Fractal Geometry
Winter Semester 2010/2011

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October 27, 2010

Exercise Set 1

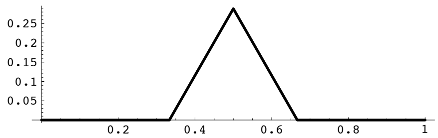
1. Show that the function $d_{\mathcal{H}} : \mathcal{H}(\mathsf{X}) \times \mathcal{H}(\mathsf{X}) \to \mathbb{R}$ defined by

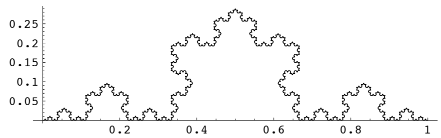
$$d_{\mathcal{H}}(A, B) := \max \left\{ \max_{a \in A} \min_{b \in B} d(a, b), \max_{b \in B} \min_{a \in A} d(b, a) \right\}$$

is a metric on $\mathcal{H}(\mathsf{X})$.

2. Show that the metric space $(\mathcal{H}(\mathsf{X}), d_{\mathcal{H}})$ is complete.

3. Find an IFS that generates the so-called *Koch curve* $\mathfrak{K}$ shown on the right-hand side of the figure below:





(*Hint: Map the unit interval onto the generator triangle. The vertices of the generator triangle are* $(\frac{1}{3}, 0)$, $(\frac{1}{2}, \frac{\sqrt{3}}{6})$, *and* $(\frac{2}{3}, 0)$.

4. Consider the IFS $\left(\mathbb{R}; \left\{\frac{1}{2}x, \frac{1}{4}x + \frac{1}{4}, \frac{1}{4}x + \frac{3}{4}\right\}\right)$. Describe the fractal set $\mathfrak{F}$ generated by this IFS. How does $\mathfrak{F}$ differ from the Cantor set?

5. Consider the IFS

$$\left(\mathbb{R}^2; \left\{ \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \right\} \right).$$

Let $A_0 := \{(\frac{1}{2}, y) \,:\, 0 \le y \le 1\}$ and let F be the set-valued map associated with this IFS.

(a) Compute and plot $A_n := F^n(A_0)$ for $n = 1, 2, 3$.

(b) Show that the attractor $A = \lim_{n \to \infty} F^n(A_0)$ is given by

$$A = \{(x, y) \,:\, y = x,\, 0 \le x \le 1\}.$$