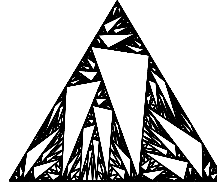
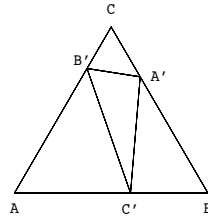


Exercise Set 2

1. Consider the figure below on the left where $\{A, B, C\} \cap \{A', B', C'\} = \emptyset$. Define ratios $r := \frac{AB'}{AC}$, $s := \frac{BC'}{BA}$, and $t := \frac{CA'}{CB}$. Define an IFS $(\Delta(A, B, C); \{f_1, f_2, f_3\})$ by requiring



that $(A, B, C) \xrightarrow{f_1} (C', A, B')$, $(A, B, C) \xrightarrow{f_2} (C', A', B)$, and $(A, B, C) \xrightarrow{f_3} (C, A', B')$. Choosing $A := (0, 0)$, $B := (1, 0)$, and $C := (0.5, 1)$, derive formulae for the maps f_1 , f_2 , and f_3 . The attractor $\mathfrak{S}(r, s, t)$ is a three-parameter family of Sierpiński triangles. A representative of this family is depicted on the right.

2. Let (X, d) be a complete metric space and let $f : [0, 1] \times X \rightarrow X$ be a parameterized family of contractions on X , i.e., each $f(t, \bullet)$, $t \in [0, 1]$, is a contraction on X . Assume that for fixed $x \in X$, f is continuous on $[0, 1]$. Prove that the fixed point x_f^* of f depends continuously on t , that is, $x_f^* : [0, 1] \rightarrow X$ is continuous.
3. Let (X, d) be a complete metric space, let $1 < N \in \mathbb{N}$, and let $f_i : [0, 1] \times X \rightarrow X$, $i = 1, \dots, N$, be a family of contractions on X depending continuously on a parameter $t \in [0, 1]$. Show that the mapping $F : [0, 1] \times \mathcal{H}(X) \rightarrow \mathcal{H}(X)$ defined by

$$F(t, A) := \bigcup_{i=1}^N f_i(t, A)$$

is continuous in t (in the metric space $(\mathcal{H}(X), d_{\mathcal{H}})$).

4. Combining the previous two problems, prove the following: Let (X, d) be a complete metric space and let $(X, \{f_1, \dots, f_N\})$ be an IFS whose mappings $f_i : [0, 1] \times X \rightarrow X$, $i = 1, \dots, N$ depend continuously on the parameter $t \in [0, 1]$. Then the attractor $\mathfrak{A}(t)$ of this IFS also depends continuously on $t \in [0, 1]$, with respect to the Hausdorff metric $d_{\mathcal{H}}$.
5. Consider the IFS $(\mathbb{C}, \{f_1 = \lambda(\bullet) - 1, f_2 = \lambda(\bullet) + 1\})$, $\lambda \in \mathbb{C}$. Show that for $\lambda = \frac{1}{2}(1 + i)$, the fractal set generated by this IFS is the famous *twin dragon*. What happens with the attractor when $\frac{1}{2}$ is replaced by α with $0 < \alpha < \frac{1}{2}$?