Fractal Geometry
Winter Semester 2010/2011

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Exercise Set 3

1. Let (M_1, d_1) be a compact metric space, (M_2, d_2) and (M_3, d_3) arbitrary metric spaces. Suppose that the mapping $f : (\mathsf{M}_1, d_1) \to (\mathsf{M}_2, d_2)$ is onto and continuous. Furthermore, assume that $g : (\mathsf{M}_2, d_2) \to (\mathsf{M}_3, d_3)$ is such that $g \circ f : (\mathsf{M}_1, d_1) \to (\mathsf{M}_3, d_3)$ is continuous. Prove that $g : (\mathsf{M}_2, d_2) \to (\mathsf{M}_3, d_3)$ is also continuous.

2. Show that (Σ_N, d_F) is a compact metric space.

3. Prove that (Σ_N, d_F) is metrically equivalent to a totally disconnected Cantor subset of $[0, 1]$.

4. Show that the code space (Σ_N, d_F) is the closure of the set of periodic codes.

5. Suppose that $\mathfrak{A}$ is the fractal set generated by the IFS $(X, \mathcal{F})$, where $\mathcal{F} := \{f_1, \ldots, f_N\}$. A point $\mathfrak{a} \in \mathfrak{A}$ is called a *periodic point* of the IFS $(X, \mathcal{F})$, if there exists a $p \in \mathbb{N}$ and a code σ of length p such that
$$\mathfrak{a} = f_{\sigma(p)}(\mathfrak{a}).$$
Conclude from Problem 4 above, that the fractal set generated by an IFS is the closure of its periodic points.