Fast Kernel Based Image Reconstruction for Photoacoustic Tomography

Principal of Photoacoustic Tomography

In Photoacoustic tomography (PAT) a short laser pulse triggers a thermoelastic expansion and contraction of the tissue. The resulting acoustic signal is recorded by transducers distributed on an acquisition surface $\partial \Omega$. PAT aims to produce an image of the initial acoustic pressure $f$ from knowledge of the recorded data. The image can reveal the pathological condition of the tissue and can facilitate a wide-range of diagnostic tasks [2].

![Figure 1: Photoacoustic effect with a partially surrounding acquisition surface.](image)

Problem Formulation

The determination of $f$ requires a solution of an inverse problem which, under certain assumptions [1], essentially amounts to the inversion of the spherical Radon transform

$$
\mathcal{M}_{\partial \Omega}(f)(\xi, t) := \frac{1}{w_{d-1}} \int_{S^{d-1}} f(\xi + tu) \, d\sigma(u), \quad \xi \in \partial \Omega, \ t \geq 0.
$$

In experimental scenarios the PAT devices constrain the recorded data to be discrete. The reconstruction problem is then, for given spherical mean data $\mathcal{M}_{\partial \Omega}(f)(\xi_k, t_l) =: g_{kl}$, to find an approximation $f^+$ of the unknown function $f$ such that

$$
\mathcal{M}_{\partial \Omega}(f^+)(\xi_k, t_l) = g_{kl}, \quad k = 1, \ldots, K, \ l = 1, \ldots, L.
$$

Kernel Based Reconstruction

An innovative kernel based reconstruction method [4], based on the concept of scattered data approximation [3], assumes that an approximation of $f$ can be given as

$$
f^+ := \sum_{i=1}^{N} \alpha_i \Phi(\cdot, y_i), \quad y_i \in \mathcal{Y}.
$$
Φ is a given positive definite radial kernel function and \( \mathcal{Y} := \{ y_i \}_{i=1}^{N} \subseteq \mathbb{R}^d \) is an arbitrary, but fixed, set of reconstruction points. Inserting (3) into (2) yields the determining equation for the coefficient vector \( \alpha \in \mathbb{R}^N \)

\[
M \alpha = g, \quad (4)
\]

where the components of the \( K \cdot L \times N \) matrix \( M \) have the form

\[
(M)_{kl,i} := M_{\partial \Omega}(\Phi(\cdot, y_i))(\xi_k, t_l). \quad (5)
\]

The problem (2) amounts then to solve the matrix vector equation (4) in a numerical stable, accurate and efficient way.

**Contributions of the Method**

1. Derivation of error-estimates for the residual \( f - f^+ \) and convergence results.

2. Analytical description of the matrix components (5) avoiding the application of error-causing quadrature rules.

3. Fast implementation of the method by the usage of reconstruction points \( \mathcal{Y} \) adapted to the transducer locations \( \{ \xi_k \}_{k=1}^{K} \).

4. Independency of geometrical restrictions of the data acquisition.

5. Application to experimental data.

Figure 2: Shepp-Logan head phantom (left) and its spherical mean values (right) for \( L = 500 \) measurement times and \( K = 360 \) transducer locations uniformly distributed on the unit circle \( S^1 \).
Figure 3: Reconstructed Shepp-Logan head phantom (left) and cross-section plot (right) through horizontal pixel line 140. The phantom was reconstructed in 387.4s with a Gaussian kernel function $\Phi(x, y) := e^{-\epsilon^2|x-y|^2}$ on a polar reconstruction grid $\mathcal{Y}$.

References


