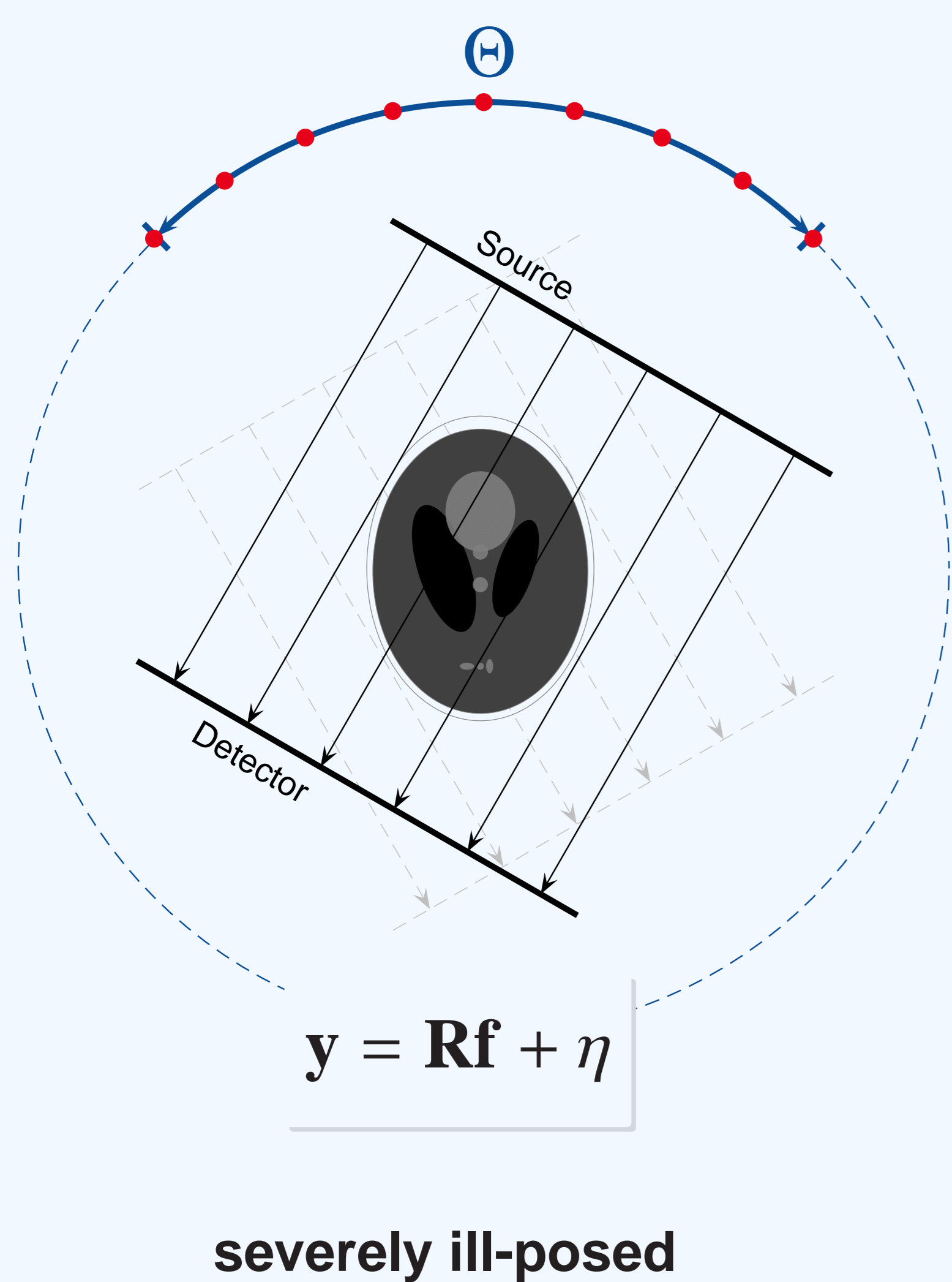


Limited Angle Tomography with Angular Undersampling

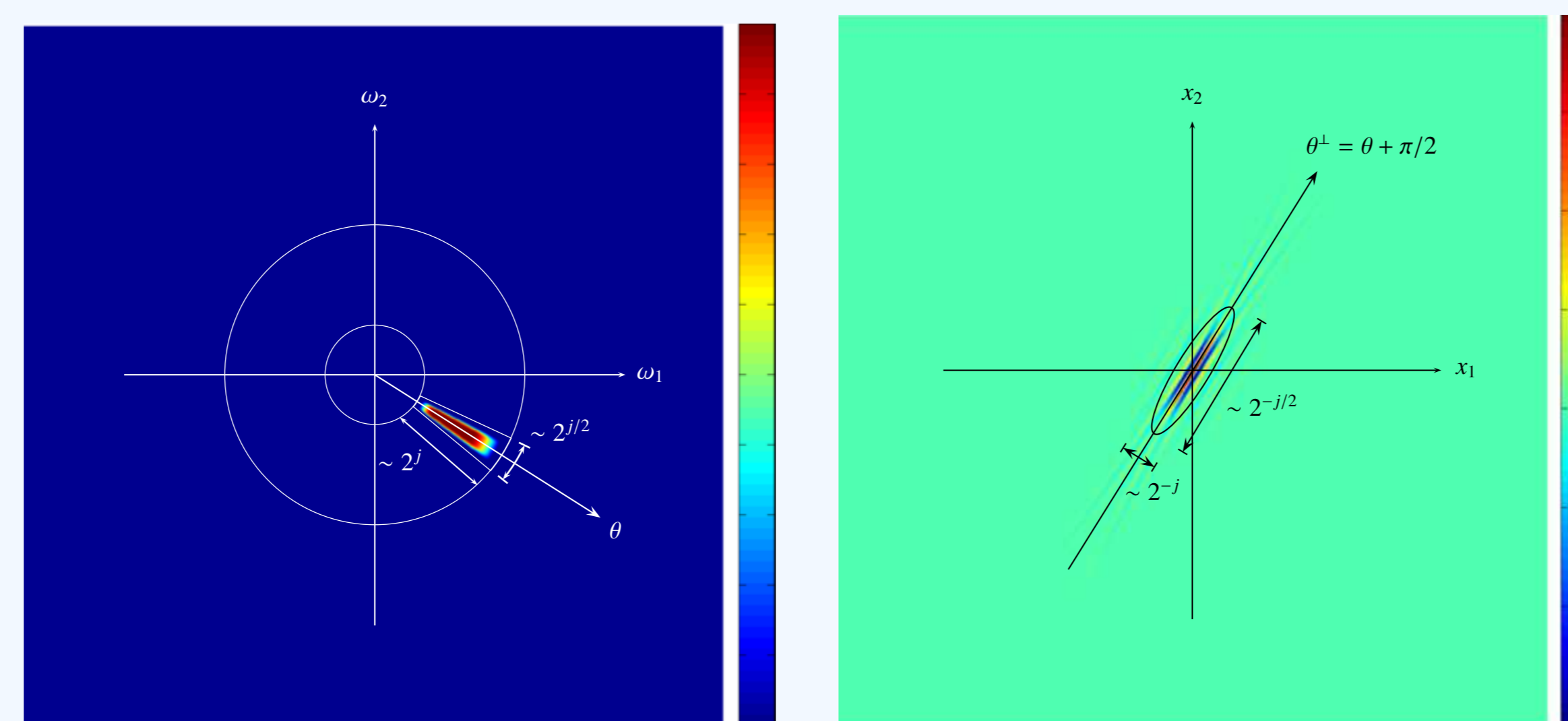


Regularization by Sparsity, [1]

$$\mathbf{x}_\alpha = \arg \min_{\mathbf{x}} \|\mathbf{R}\Psi \cdot \mathbf{x} - \mathbf{y}\|_2^2 + \alpha \|\mathbf{x}\|_1$$

Ψ is a dictionary with $f = \Psi \cdot x$; \mathbf{x}_α has few non-zero entries

Sparse Representation and Adaption by Curvelets γ_μ



$$f = \underbrace{\sum_{a,b} \sum_{\theta \in \Theta} x_{\mu(a,b,\theta)} \gamma_{\mu(a,b,\theta)}}_{=\Gamma_\Theta \cdot \mathbf{x}} + \underbrace{\sum_{a,b} \sum_{\theta \in [0,\pi] \setminus \Theta} x_{\mu(a,b,\theta)} \gamma_{\mu(a,b,\theta)}}_{\text{invisible}}$$

I. + II.

III.

New Framework

I. Regularization

II. Resolve Undersampling

III. Adaption to Limited Angle Geometry

Visible Singularities (Boundaries) from Limited Angle Data Set

A singularity is **visible** iff there is an **x-ray** which is **tangent to its boundary**, [2].



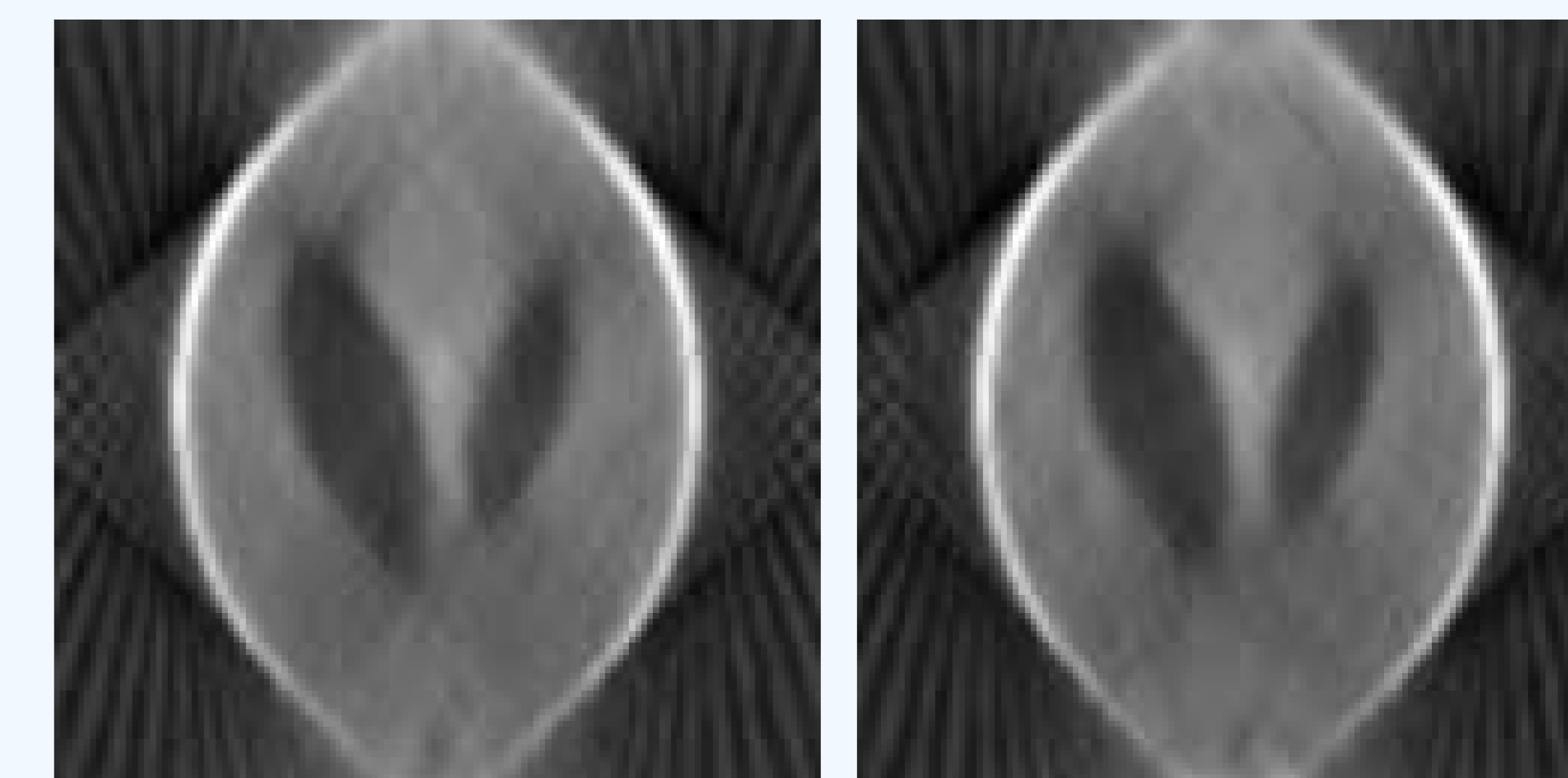
Visible singularities of Head-Phantom
Angular range $\Theta = [45^\circ, 135^\circ]$

New Method Adapted Sparse Regularization for Limited Angle Tomography

1. Determine visible directions Θ
2. Formulate the problem in the curvelet domain with respect to visible directions, $y = \mathbf{R}\Gamma_\Theta \cdot x + \eta$
3. Compute visible curvelet coefficients $x_{\alpha,\Theta}$ via

$$\mathbf{x}_{\alpha,\Theta} = \arg \min_{\mathbf{x}} \|\mathbf{R}\Gamma_\Theta \cdot \mathbf{x} - \mathbf{y}\|_2^2 + \alpha \|\mathbf{x}\|_1$$
4. Compute reconstruction $f_{\alpha,\Theta} = \Gamma_\Theta \cdot x_{\alpha,\Theta}$

Speedup of the Sparse Regularization while Preserving the Reconstruction Quality



Full Sparse Regularization
I.+II.

Adapted Sparse Regularization
I.+II.+III.

Dimensionality reduction in the curvelet domain by $\sim 50\%$
Speedup of reconstruction time by $\sim 40\%$

References

- [1] Ingrid Daubechies et al., "An iterative thresholding algorithm for linear inverse problems with a sparsity constraint," *Comm. Pure Appl. Math.*, vol. 57, no. 11, pp. 1413–1457, 2004.
- [2] Eric Todd Quinto, "Singularities of the X-ray transform and limited data tomography in \mathbb{R}^2 and \mathbb{R}^3 ," *SIAM J. Math. Anal.*, vol. 24, no. 5, pp. 1215–1225, 1993.