A NEW FRAMEWORK FOR SPARSE REGULARIZATION IN LIMITED ANGLE X-RAY TOMOGRAPHY

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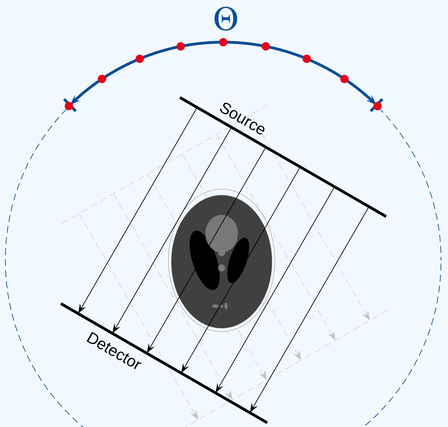
Limited Angle Tomography with Angular Undersampling



$$\mathbf{y} = \mathbf{R}\mathbf{f} + \eta$$

severely ill-posed

New Framework

I. Regularization

II. Resolve Undersampling

III. Adaption to Limited Angle Geometry

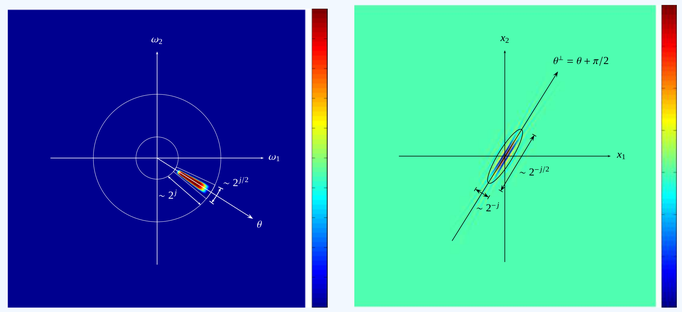
Regularization by Sparsity, [1]

$$\mathbf{x}_\alpha = \arg\min_{\mathbf{x}} \|\mathbf{R}\boldsymbol{\Psi}\cdot\mathbf{x} - \mathbf{y}\|_2^2 + \alpha\,\|\mathbf{x}\|_1$$

$\boldsymbol{\Psi}$ is a dictionary with $f = \Psi \cdot x$; $\mathbf{x}_\alpha$ has few non-zero entries

I. + II.

Sparse Representation and Adaption by Curvelets γ_μ



Frequency domain — Spatial domain

$$f = \underbrace{\sum_{a,b}\sum_{\theta\in\Theta} x_{\mu(a,b,\theta)}\gamma_{\mu(a,b,\theta)}}_{=\boldsymbol{\Gamma}_\Theta\cdot\mathbf{x}} + \underbrace{\sum_{a,b}\sum_{\theta\in[0,\pi)\setminus\Theta} x_{\mu(a,b,\theta)}\gamma_{\mu(a,b,\theta)}}_{\textbf{invisible}}$$

III.

Visible Singularities (Boundaries) from Limited Angle Data Set

A singularity is **visible** iff there is an **x-ray** which is **tangent to its boundary**, [2].

Visible singularities of Head-Phantom
Angular range $\Theta = [45°, 135°]$

New Method

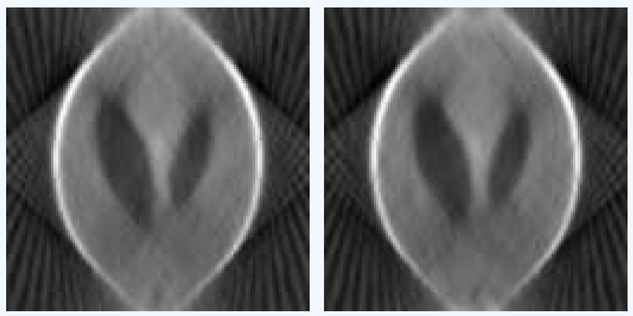
Adapted Sparse Regularization for Limited Angle Tomography

1. Determine visible directions Θ
2. Formulate the problem in the curvelet domain with respect to visible directions, $y = R\Gamma_\Theta \cdot x + \eta$
3. Compute visible curvelet coefficients $x_{\alpha,\Theta}$ via

$$\mathbf{x}_{\alpha,\boldsymbol{\Theta}} = \arg\min_{\mathbf{x}} \|\mathbf{R}\boldsymbol{\Gamma}_{\boldsymbol{\Theta}}\cdot\mathbf{x} - \mathbf{y}\|_2^2 + \alpha\,\|\mathbf{x}\|_1$$

4. Compute reconstruction $f_{\alpha,\Theta} = \Gamma_\Theta \cdot x_{\alpha,\Theta}$

Speedup of the Sparse Regularization while Preserving the Reconstruction Quality



Full Sparse Regularization I.+II. — Adapted Sparse Regularization I.+II.+III.

Dimensionality reduction in the curvelet domain by $\sim 50\%$
Speedup of reconstruction time by $\sim 40\%$

References

[1] Ingrid Daubechies et al., "An iterative thresholding algorithm for linear inverse problems with a sparsity constraint," *Comm. Pure Appl. Math.*, vol. 57, no. 11, pp. 1413–1457, 2004.

[2] Eric Todd Quinto, "Singularities of the X-ray transform and limited data tomography in $\mathbb{R}^2$ and $\mathbb{R}^3$," *SIAM J. Math. Anal.*, vol. 24, no. 5, pp. 1215–1225, 1993.

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GE imagination at work

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