Individual Grants Programme

Renewal for the proposal:

Robust dynamic programming approach to aircraft control problems with disturbances

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Project Description

1 State of the art and preliminary work

Aircraft control is a highly complex and safety critical field, which even today poses major challenges to ensure safe operational conditions. In this context, disturbances such as wind gusts or measurement errors represent major threats and need to be addressed carefully during the whole development phase. This applies especially to the most critical flight phases, such as departure or landing, where robust performance of flight control systems is of paramount importance.

In the past, great effort has been invested into the development of reliable control concepts in order to ensure robust disturbance rejection and guarantee safe operation within the whole flight envelope. Before a control law is cleared for flight, it has to be shown that safety boundaries are not breached and performance metrics are robustly achieved. Thus, besides the design task, testing procedures of these flight control laws represent a highly relevant topic of research. Both tasks, robust control design and testing with respect to disturbances, can be addressed by differential game and open loop optimal control theory and are the main focus of this project proposal.

In the following we present a brief overview of the state of the art for differential game and optimal control theory for the application to robust aircraft control. The primary focus will be on the main findings and the preliminary results of the ongoing joint DFG project Robust dynamic programming approach to aircraft control problems with disturbances involving the Institute of Flight System Dynamics (FSD) and the Mathematical Modeling Group (MMG) at the Technical University of Munich (TUM). Note that this project has been extended by the project grant pr74lu of the Gauss Centre for Supercomputing (see https://www.lrz.de/projekte/hlrb-projects/) in order to facilitate the solution process of high dimensional differential games on large scale grid computers.

Differential game theory

This section outlines basic notions of differential game theory such as value function and viability kernel. The description is given in terms of general nonlinear differential games formalized according to [31].

Consider the following differential game:

\[ \dot{x} = f(t, x, u, v), \quad x \in \mathbb{R}^n, \quad u \in P \subset \mathbb{R}^p, \quad v \in Q \subset \mathbb{R}^q, \]

(1)

where \( u \) and \( v \) are control inputs of the first and second player, respectively. The sets \( P \) and \( Q \) are given compacts. The game starts at \( t_0 \in [0, T] \) from \( x_0 \in \mathbb{R}^n \) and finishes at \( T \). The aim of the first (resp. second) player is to minimize (resp. maximize) an objective functional of the form:

\[ J(x(\cdot)) = \max \left\{ \sigma_0(x(T)), \max_{\tau \in [t_0, T]} \sigma(\tau, x(\tau)) \right\}, \]

(2)

where \( \sigma_0 : \mathbb{R}^n \to \mathbb{R} \) and \( \sigma : [0, T] \times \mathbb{R}^n \to \mathbb{R} \) are given functions satisfying the condition \( \sigma_0(x) \geq \sigma(T, x), \quad x \in \mathbb{R}^n \).

In formula (2), the first term in braces evaluates the state vector at the termination time instant \( T \), whereas the second term accounts for time dependent state constraints, see [19].
The value function is defined by the relation 

\[ \forall \psi \in \mathcal{P} \quad V(t_0, x_0) = \max_{\nu} \min_{\mu} J(x(\cdot)) = \min_{\psi} \max_{\nu} J(x(\cdot)), \]

where the minimum is taken over feedback strategies of the first player, whereas the maximum is computed over the so-called feedback counter strategies of the second player (see [31]). This means that the second player (e.g., wind) can measure the current choice of the first player (e.g., the power setting), which makes the second player more dangerous. Note that involving counter strategies is necessary because the following saddle point (Isaacs) condition:

\[ \min_{u \in P} \max_{v \in Q} \langle \ell, f(t, x, u, v) \rangle = \max_{v \in Q} \min_{u \in P} \langle \ell, f(t, x, u, v) \rangle, \quad \ell \in \mathbb{R}^n, \quad x \in \mathbb{R}^n, \quad (3) \]

is generally not fulfilled for aircraft models.

**Computing value functions.** It is known that the value function is a special viscosity solution of the following Hamilton-Jacobi equation (see [27, 45, 19]):

\[ V_t + H(t, x, V_x) = 0, \quad V(T, x) = \sigma_0(x), \quad \text{where} \quad H(t, x, p) = \min_{u \in P} \max_{v \in Q} \langle f(t, x, u, v), p \rangle. \quad (4) \]

Viscosity solutions of (4) can be computed using monotone grid schemes proposed in [19] and [20]. Other methods based on Lax-Friedrichs numerical Hamiltonians can be found in [35] and [36].

The application of differential games to control of aircrafts can be found, e.g., in [39, 16], see also [1, 2, 7, 8].

**Viability approach.** If the aim of the control is to keep the system inside a state constraint, the maximal viable subset, viability kernel, of the state constraint has to be found, see [15]. Moreover, this yields a feedback control that produces trajectories remaining in the viability kernel, and hence in the state constraint, for all admissible disturbances. It should be emphasized that the notion of viability kernel is typical for control systems. In the case of differential games, the terms discriminating and leadership kernels are more suitable, see e.g., [25]. The discriminating kernel corresponds to the case where the first player (pilot) can exactly measure current wind components to use counter feedback strategies. In contrast, the leadership kernel assumes that the second player (wind) knows current controls of the pilot and uses feedback counter strategies, which is rather realistic in context of computing guaranteeing controls. If the condition (3) holds, then discriminating and leadership kernels coincide.

Assume that the conflict control system (1) is autonomous, \( \sigma_0 \equiv 0 \), and \( \sigma \) is time independent. It is proven (see [21, 4, 11]) that there exists a limiting function \( \psi(x) = \lim_{t \to -\infty} V(t, x) \) possessing the following property: For each \( \lambda \in \mathbb{R} \), the set \( \{ x : \psi(x) \leq \lambda \} \) is the leadership kernel of the state constraint \( \{ x : \sigma(x) \leq \lambda \} \). A feedback control keeping the system in the leadership kernel can be found from the principle of maximum descent of the function \( \psi \).

Papers [21, 4, 11] propose a grid method for approximate computing the function \( \psi \). The method is based on the iterative application of a monotone grid operator. A comprehensive overview of formal algorithms for finding viability, discriminating, and leadership kernels can be found in [37]. Possible numerical implementations are also discussed there.

The application of viability approach to control of aircrafts and other plants can be found in [42, 43, 38], see also [11, 12, 13].
Optimal control theory

Optimal control theory provides the theoretical background for solving the following problem class \[17, 29\]: Find the optimal state trajectories \( \hat{x}(t) \), control histories \( \hat{u}(t) \), and parameters \( \hat{p} \) for the dynamic system \( \dot{x}(t) = f(x(t), u(t), p, t) \) with respect to the general BOLZA cost function

\[
J = e(x(t_0), x(t_f)) + \int_{t_0}^{t_f} L(x(t), u(t), p, t) \, dt.
\]  

Additionally, the solution has to satisfy path constraints \( c(x(t), u(t), p, t) \leq 0 \) and initial/final boundary conditions \( \phi(x(t_0), x(t_f)) = 0 \) at the initial time \( t_0 \) and final time \( t_f \). Popular methods for solving this type of problem formulation are indirect approaches, function space methods, direct methods, and dynamic programming.

Open loop optimal control for aircraft trajectory optimization in the presence of wind has been studied in \[33, 34\]. The wind field is prescribed, and optimal ascent trajectories are computed by solving appropriate trajectory optimization problems. Furthermore, the articles \[22, 23\] study abort landing as a minimax optimal control problem. Based on the reformulation of the problem, a numerical solution is achieved by multiple shooting in combination with a homotopy procedure. Optimal escape trajectories in the presence of micro-burst wind shears are investigated in \[40\]. Moreover, \[26\] presents robust take-off strategies with different wind shear intensities.

In addition, the application of optimal control methods for safety assessment has been studied in the past: An automatic landing procedure for a hybrid aircraft system with mode changes due to flap deployment during the approach is presented in \[16\]. The set of safe states is computed in addition to an optimal control law that keeps the aircraft within this set. Article \[30\] motivates the application of optimal control theory for testing flight control laws with respect to continuous inputs. Physical restrictions are included in the optimal control formulation using state constraints.

Joint preliminary work of the Institute of Flight System Dynamics and the Mathematical Modelling group

Control on a finite time interval

Here, we refer to differential games considered on finite time intervals. Such problem formulations are appropriate for the description of certain flight phases such as runway acceleration, take-off, landing, abort landing, etc.

Paper \[1\] describes a highly nonlinear model of aircraft lateral runway motion under wind disturbances. The players, autopilot and wind, strive to minimize, respectively, maximize the cost functional of the form \[2\], which makes it possible to account for state constraints imposed on the state variables \( x_1, x_2, \ldots, x_n \). The model parameters approximately correspond to a Boeing-727 aircraft. Optimal feedback controls, which can work against side wind gusts of 17 m/s, are constructed using grid methods described in \[18\] and \[19\]. It should be noted that the sparse grid approximation technique, see e.g. \[46\], has been tested in \[1\].

Paper \[2\] concerns with the abort landing problem in the framework of differential game theory. The novelty of this paper consists in the application of numerical methods described in \[18\] and \[19\] to a nonlinear aircraft model with state constraints.

Papers \[7\] and \[8\] consider take-off problems in windshear with reference to flight in the vertical plane. A point-mass model of a generic modern regional jet aircraft is used. The
following technique of sequential linearization is applied. The time interval of the process is divided into short time subintervals, and the aircraft dynamics is linearized at the beginning of each subinterval around a straightforward equilibrium trajectory providing a good climb and preventing, whenever possible, the aerodynamic velocity from decreasing. The linearized dynamics are considered on the current time subinterval as a differential game where the first player (autopilot) strives to keep the state of the linearized dynamics close to zero, so that the state vector of the nonlinear dynamics follows, as far as possible, the straightforward equilibrium linearization trajectory. The second player (wind) plays in the linearized dynamics against the autopilot. A feedback control found from the linearized dynamics is used in the nonlinear case on the current time subinterval. Then, the procedure is being repeated for the next time subinterval. It should be noted that a fast method for approximate solving auxiliary linearized differential games on the time subintervals is used. Thus, the procedure can be implemented as a real-time control scheme.

Paper [10] presents quite new results which are supposed to be supplied as a sample chapter for our prospective book in the Springer series “SpringerBriefs in Control, Automation and Robotics”. The publication deals with the generation of dangerous disturbances in problems of aircraft control. The method is based on the construction of the so-called repulsive tubes in linear differential games using a dynamic programming method. The computation involves solving a large number of linear programs. For this purpose, a fast algorithm for low-dimensional linear programming is developed. The method can be applied to nonlinear aircraft models using a sequential linearization of the model.

**Viability approach and large scale grid computing**

In the following papers, a grid method is used to compute viability (more exactly, discriminating and leadership) kernels for different problems of aircraft control under windshear conditions.

Paper [4] describes a grid method for the numerical computation of leadership and discriminating kernels, which are referred to in the paper as minimax and maximin viability kernels, respectively. The method is applied to a point-mass model describing a generic modern regional jet transport aircraft flying in a vertical plane. The cruise phase (flight at the established level with practically constant configuration and speed) in the presence of wind gusts is considered. Discriminating/leadership kernels (they coincide for the considered model) are computed in five-dimensional space, and optimal feedback controls are found.

Paper [11] develops the description of discriminating/leadership kernels through minimal \( u \)-stable (see [31]) lower semi-continuous functions. This substantiates the grid algorithm for the computation of these kernels. The developed theory and numerics are applied to a point mass model accounting for both vertical and lateral motion of a generic modern regional transport aircraft. The corresponding leadership kernel is computed in five-dimensional space, optimal feedback controls are found, and trajectories (remaining according to the theory in the leadership kernel) are simulated.

Paper [12] considers the same aircraft model as in [4]. However, this model is extended to seven state variables in order to add filters smoothing control inputs and wind gusts. Discriminating/leadership kernels (they coincide for the model considered) are computed in five-dimensional space, and optimal feedback controls are found. The paper outlines the data flux between compute nodes and processors of the SuperMUC system at the Leibniz Supercomputing Centre of the Bavarian Academy of Sciences and Humanities.
Paper [13] addresses the analysis of an adaptive aircraft controller (see [5]) using viability theory. The application of viability theory allows us to prove that the controller can successfully work against worst-case uncertainties in parameters, even in the case where these uncertainties are active time dependent disturbances. Moreover, the viability technique is also applicable to the case where the parameters defining the learning rate of the controller are constants rather than time dependent controls. The existence of a viable subset of state constraints means that the internal feedback channel of the controller can successfully work against even time-dependent worst-case uncertainties.

It should be mentioned that the numerical construction of discriminating/leadership kernels is very time and memory consuming because the computation involves an iterative procedure applied to multivariate grid functions. To implement such algorithms, the applicants use computer resources of the Gauss Centre for Supercomputing/Leibniz Supercomputing Centre under grant: pr74lu (see http://www-m6.ma.tum.de/~botkin/SuperMuc.pdf for the application to this grant).

The viability control approach is currently being integrated in the realistic flight simulator at FSD. One of the major challenges is that the viability kernel can be constructed only for a low number of states (curse of dimensionality). Therefore, we decouple the control structure by defining two distinct control loops. This separation of loops is a very common concept in flight control and is based on the assumption that the translational dynamics (outer-loop) is much slower than the rotational dynamics (inner-loop). This implies that the translational dynamics can be seen as quasi steady state from the viewpoint of the rotational dynamics. Thus, in our application, the viability kernel is constructed based on a model for the translational and attitude dynamics (outer-loop) and the trajectory generated thereof is tracked by the rotational dynamics (inner-loop) (see Fig. 1). The states for the viability kernel computation are the translational states (kinematic velocity, kinematic climb angle, and kinematic course angle) and a first order low pass model for the attitude states (kinematic angle of attack, kinematic bank angle, and kinematic sideslip angle). The main concept of the inner-loop is to use a control architecture based on the principle of Nonlinear Dynamic Inversion (NDI) with linear reference models [44]. This inner-loop control structure ultimately produces actuator inputs such that the corresponding states of the full model follow the reference model states. Note that the use of the same attitude reference model (kinematic angle of attack, kinematic bank angle, and kinematic sideslip angle) for the NDI based inner-loop and the construction of the viability kernel facilitates tracking of the robust viable solution.

Flight control law integration and testing

The thorough testing of flight control laws regarding disturbance rejection and the effect of modeling uncertainties is of paramount importance for flight safety. Classical control structures are usually assessed by established metrics such as gain or phase margins and the requirements regarding properties of linearized systems. Moreover, flight control law testing is performed over the whole flight envelope. Due to the novel control approach investigated in the ongoing project, testing procedures need to be employed, which are specifically tailored to take into account continuous disturbances (wind).

Therefore, two approaches for testing flight control laws are being developed: The first one is based on a special case of differential game theory, which provides optimal worst cases for simplified models. The second is based on an appropriate optimal control formulation and provides suboptimal worst cases for high dimensional models. In both approaches the
problem is formulated between the aircraft closed loop system (dynamics and controller) and the continuous disturbance and can be stated as follows: Find the worst case disturbance inputs that violate the performance or safety criterion. The novel testing approaches were developed using benchmark examples featuring the distinct parts of the control structure under development.

In [3] the corresponding optimal control formulation is presented. The criterion to be tested is introduced in the cost function (5), and physical bounds, i.e. actuator position and rate limits, are accounted for via state constraints. It was shown that, for linearized models with state constraints, the worst case solution for continuous inputs such as wind is bang-bang or bang-singular-bang depending on the switching function. The numerical results were obtained for a closed loop model linearized around a horizontal steady state flight condition. Note that in this flight condition the linearized model decouples and allows for the separate treatment of the longitudinal and lateral motion.

The corresponding nonlinear case is studied in [9]. The approach herein extends the formulation for testing flight control criteria using a combination of continuous disturbances (wind, worst case control inputs) and parameter uncertainties. This is achieved by constraining the set of admissible parameters to the confidence interval of a multivariate Gaussian distribution obtained from a maximum likelihood estimator. The proposed method shows promising results and seems to be highly effective in finding worst-case disturbances.

In [6], a homotopy procedure is presented to test the performance of an incremental nonlinear controller with respect to continuous wind disturbances. Note that the generation of good initial guesses is of high importance when solving optimal control problems via gradient based optimization algorithms. Here, the initial guess is generated using a sequence of
different methods. First, candidate solutions are generated via Latin Hypercube sampling in a low dimensional space. Subsequently, these solutions are refined by a global optimization scheme (differential evolution). The best solutions are then taken as the initial guess for the discretized and thus high dimensional direct optimal control problem. Furthermore, first and second order post-optimal sensitivity analysis was used to assess the influence of model parameters on the worst case solution.

In [5], a model reference adaptive controller is tested with respect to continuous inputs and parameter uncertainties. This approach is based on the construction of the reachable set in the tracking error subspace via the Distance Field on Grids Method [24]. This method formulates appropriate optimal control problems such that the feasible set coincides with the reachable set and thus allows for an estimation of the maximum tracking errors.

1.1 Project-related publications

1.1.1 Articles published by outlets with scientific quality assurance, book publications, and works accepted for publication but not yet published


1.1.2 Other publications

None.

1.1.3 Patents

None.

1.1.3.1 Pending

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1.1.3.2 Issued

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2 Objectives and work programme

2.1 Anticipated total duration of the project

The ongoing DFG project *Robust dynamic programming approach to aircraft control problems with disturbances* started in March 2015 (MMG) and June 2015 (FSD) and has a total duration of three years. The anticipated duration of the prolonged project is three years. DFG funds will be necessary during the whole time.

2.2 Objectives

Novel conflict control problem formulations can be used to robustify baseline control structures. For example, a conflict control problem may be formulated as a game between parameters (e.g. gain values) of a baseline controller (first player) and disturbances of different kinds (second player). Referring to that, first experiments have been conducted in [13]. Here, we allowed the first player to vary parameters (learning rates) of an adaptive controller within predefined bounds. This ansatz showed to be effective for counteracting continuous disturbances. Such a method represents a promising approach to combine a wide class of conventional control concepts with robust approaches by means of differential game theory.

The main objective of the prolonged project is an extension of differential game theory and its numerical methods towards designing a robust trajectory controller integrated in the control architecture illustrated in Fig. 2.

![Figure 2: Extension of the robust aircraft control concept with a differential game based baseline control law adaptation (schematic).](image-url)
It should be noted that a control structure for the inner-loop has already been developed for the case of tracking viable trajectories produced by feedback controls determined from viability kernels (see Section 1). This inner-loop control structure will now be extended by an outer-loop baseline trajectory controller. It should be designed to exhibit good command tracking characteristics, whereas the adaptation based on the differential game approach provides good disturbance rejection. Note that the role of differential games here is different from that described in Section 1. Now, feedback controls obtained from viability kernels have to affect parameters (e.g. gain values) of the controller to compensate disturbances (e.g. wind gusts) trying to push the aircraft off the desired trajectory. This approach will be tested for typical phases of flight such as departure, holding, and landing. Note that especially in these phases of flight, wind gust rejection is of paramount importance for flight safety. Furthermore, the effect of measurement errors on the trajectory controller is crucial to account for noisy or inaccurate sensor data. Therefore, we plan to additionally include these disturbances in the differential game formulation.

It should be noted that the above formulated problems require the enhancement of theoretical results and numerical methods related to the construction of viability kernels. The main challenges here are huge computational efforts leading to large computation times even on large scale grid computers. In this connection, we need to speed up the convergence of our iterative algorithms for computing viability kernels. This can be done by utilization of sparse representation of value functions and by adopting policy iteration algorithms proposed in [14]. The first aim here is to extend our algorithms for computing viability kernels up to 9 dimensions.

Concerning trajectory tracking, it is worth to mention a differential game theoretic method which is based on direct aiming to $u$-stable trajectories (see [31]). Assume that, in case the second player (disturbance) shows its constant control on a short time interval, the first player (controller) can always force the plant to meet the trajectory at the end of this time interval. Then, if the dynamics of the plant satisfy the saddle point condition (3), the aiming procedure enables to follow the trajectory without any information about the disturbance. It should be noted that the plant’s dynamics can always be slightly relaxed to fulfill the above-mentioned saddle-point condition. This method was adopted in [32] for tracking trajectories of dynamic systems.

Another objective is related to the extension of flight control law clearance methods with respect to disturbances based on differential game and optimal control theory. As outlined in Section 1 a special formulation of conflict control problems (see e.g. [10] and [13]) and open-loop methods (see e.g [3]) are suitable for testing the developed control structure with respect to worst-case inputs such as wind gusts. For obtaining globally optimal worst-case solutions, the number of disturbances and the augmented state space for the trajectory following problem will pose challenges for the implementation of the differential game solver on large scale grid computers (SuperMUC). Further investigations are necessary e.g. in the field of sparse grid function representations in order to render the numerical solution feasible. Furthermore, low-dimensional, but not over-simplified, models need to be derived that can be used for testing purposes. Particularly for the optimal control problem formulation, the local nature of solutions from gradient based optimization schemes needs to be addressed. Methods based on homotopy, which gradually build up the complexity of the problem, i.e. nonlinearity and dimensionality, are a promising candidate for the application to this specific problem type. Moreover, the computational burden associated with solving large scale optimal control problems of this type depends heavily on the exploitation of sparse structures. Especially
the efficient implementation of the transcription methods for the differential equation needs to be handled carefully.

Summarizing, the objectives for the work programme are as follows:

- Extension of the developed control structure (see Fig. 1), featuring a robust inner-loop, towards a robust outer-loop trajectory controller (see Fig. 2).
- Formulation of differential games between parameters (e.g. gains) of the baseline trajectory controller and the disturbances. Besides wind, sensor measurement errors will be included as disturbances in the formulation.
- Extension of the developed solver on large scale grid computers to handle the increased number of disturbances and the augmented state space.
- Generation of realistic reference trajectories for typical flight phases (e.g., departure, holding, landing) and development of disturbance models for testing purposes.
- Further development of testing procedures using a special type of conflict control problems and optimal control theory.
- Integration and testing of the robust trajectory controller on the flight simulator at the Institute of Flight System Dynamics.

2.3 Work programme incl. proposed research methods

The joint work programme and proposed methods for the objectives outlined in the previous section will be presented in the following. We firstly focus on the high level description of the work packages for the joint programme. The individual work share between FSD and MMG within these packages will be outlined subsequently. As the performance of the trajectory controller is highly dependent on the appropriate baseline controller that is expected to be included in the conflict control problem, the close cooperation in all parts of the work programme between FSD and MMG is of high importance.

WP 1: Development of the baseline trajectory controller
(FSD, Month 1–14) The existing inner-loop control structure is extended by a baseline trajectory controller to allow for tracking smooth reference trajectories. Thus, the baseline control architecture with the respective parameters needs to be designed. This includes models for the opposing players, i.e. sensor errors and wind disturbances. Furthermore, appropriate bounds for the baseline control parameter adaptation have to be established, which can be used for the conflict control problem.

WP 2: Enhancement of the solver on large scale grid computers
(MMG, Month 1–17) New statements of conflict control problems assume the increase of the number of control and disturbance inputs as well as the extension of the state space. A challenging task here is to enhance our algorithms for computing viability kernels to match these requirements.

WP 3: Implementation of realistic test cases (reference trajectories)
(FSD/MMG, Month 15–21) Based on realistic routes from air traffic, reference trajectories for the relevant flight phases are defined. The test scenarios should be as realistic as possible, meaning that the routes have to be defined including the constraints imposed from air traffic regulations, i.e. by Standard Instrument Departure (SID) and Standard Terminal Arrival
Route (STAR). Additionally, these trajectories are evaluated concerning their $u$-stability with respect to different plant dynamics.

**WP 4: Development of optimal and suboptimal testing procedures**

(FSD/MMG, Month 19–31) For the thorough testing of control laws, optimal and suboptimal procedures are further investigated. On the one side, the differential game formulation allows for determining optimal worst case disturbances for reduced models. On the other side, an appropriate direct optimal control formulation enables the testing of accurate high dimensional models using sub-optimal solutions. Both approaches will be extended by MMG and FSD for testing the trajectory controller with respect to continuous wind disturbances and sensor measurement errors. The major challenges for the mentioned methods are on the one hand the derivation of reduced, but not over-simplified models for the differential game approach and on the other hand the solution of non-linear high dimensional optimal control problems using direct methods.

**WP 5: Preliminary testing using simplified models**

(MMG/FSD, Month 22–31) During the development phase, the approach will be tested by means of simplified test scenarios in order to streamline the final integration in the flight simulator. This includes the different flight phases and the modeling depth of the dynamic system and control structure. The test cases should build up regarding the nonlinearity of the model and the degrees of freedom (longitudinal, lateral).

**WP 6: Integration and final testing in the flight simulator**

(FSD, Month 30–32) The robust trajectory control law needs to be integrated in the flight simulator at FSD and tested with respect to stability and performance metrics.

**WP 7: Documentation**

(FSD/MMG, Month 32–36) The overall work done and the results achieved are prepared for and documented in the final report.

The following Gantt chart illustrates the schedule of the joint work programme.

![Gantt Chart](image)

Figure 3: Overview of the work programme.

It should be stressed that the close collaboration between the mathematical modeling group and the Institute of Flight System Dynamics has so far led to a very efficient and fruitful
cooperation. Due to the different areas of expertise of the applicants, the knowledge transfer on the one side in the field of differential games and applications and on the other side aircraft modeling and modern control design has shown to be extremely beneficial for the ongoing project.

Work programme for the Mathematical Modeling Group

WP 2: Enhancement of differential game solvers and optimization of data storage

We expect that the design of the robust trajectory controller requires solving nonlinear differential games (computing viability kernels) in 9 dimensions. Note that by now we can use such a technique in the case of maximum 7 state variables, see paper [13] where an adaptive controller proposed in [5] has been investigated using the viability approach. The challenging problem is to gain two additional state space dimensions along with an increase of the number of control and disturbance inputs. This can be solved by utilizing sparse grid representations of the value functions, which assumes the frequent usage of interpolation operations on sparse grids. Our first experiments show that such operations run rather slowly even in five dimensions. Nevertheless, it is possible to accelerate them because they are being repeatedly performed at the same state space points in each step of the solution algorithm. The corresponding preliminary experiments are already conducted. A further acceleration can be achieved by adopting the so-called policy iteration algorithm proposed in [14]. Moreover, a method of fast computing the minimax operation over control and disturbance parameters, respectively, (see [31]) can be adopted.

The next task concerns the extraction of optimal feedback controls and disturbances stored in the sparse form. This process should be accelerated to achieve the real-time performance. Another development should be concerned with finding worst-case disturbances in linear differential games of high dimensions, cf. [10]. This method can be interpreted as solving a linear differential game and finding a \( v \)-stable (see [31]) upper approximation of the value function. The method is implementable on a common serial computer in space of up to 12 dimensions. It has to be enhanced and implemented on the SuperMUC system to achieve the dimensionality up to 24.

WP 3: Testing the feasibility of direct aiming methods for realistic trajectories

Testing realistic trajectories obtained in WP 3 of FSD concerning their \( u \)-stability with respect to different plant dynamics is supposed. The method of direct aiming to \( u \)-stable sets should be enhanced to exclude chattering regimes.

WP 4: Statement of reduced differential games for suboptimal testing

Here, two tasks can be formulated: First, the trajectory controller augmented by the translation dynamics, i.e. without the robust inner loop, has to be tested with respect to continuous wind disturbances and sensor measurement errors. This can be done by constructing worst-case disturbances from the viability set that is assumed to be constructed for finding a feedback strategy affecting the controller parameters (e.g. the gain values). As it was mentioned in WP 2, the corresponding differential game has 9 state variables. It should be noted that the translation dynamics can be decomposed into longitudinal and lateral parts, which allows us to consider two differential games of less dimensionalities. The disturbances found from these games can be used as suboptimal testing inputs.
Second, the above mentioned suboptimal worst-case disturbances will be used for testing the global control structure, including the robust inner loop. Additionally, a linearized model of the global structure, in its full dimensionality, will be treated with the worst-case disturbance algorithm presented in [10].

WP 5: Preliminary testing using reduced differential games

The implementation of this package assumes solving differential games formulated in WP 4 to obtain suboptimal wind and measurement disturbances. The disturbances have to be computed and stored in the form of feedback strategies. We can start with disturbances found from the longitudinal channel and then extend them by disturbances separately defined from the lateral one. In the next step, disturbances computed from the 9-dimensional differential game mentioned in WP 4 are supposed to be tested. Finally, the linearized global structure should be used to find disturbances according to the algorithm from [10]. It should be noted that the last technique is practically not restricted by the dimensionality of the global model.

WP 7: Documentation

Intermediate results obtained in course of the proposal’s implementation should be published. The overall work done and the results achieved need to be prepared for and documented in the final report.

Work programme for the Institute of Flight System Dynamics

WP 1: Development of the baseline trajectory controller

In this work package the baseline control structure for the trajectory loop is developed. The high level requirement for this controller is to follow predefined state trajectories for given values of i.e. position, velocity, course angle, and climb angle. To enable the velocity control, additionally a thrust control law needs to be implemented. In this context, the control structure has to be designed in such a way that it allows for the parameter adaptation using the differential game formulation. Besides the gain design using classical methods from control theory, appropriate bounds for the gains need to be established, which define the degrees of freedom for the adaptation in the differential game formulation. In order to test the applicability to the large range of available classical control concepts, the baseline controller design should include variants based on relevant design philosophies (such as Eigenstructure Assignment/LQR). Moreover, a robust variant (e.g. based on $H_\infty$ methods) should be included in order to compare and benchmark the differential game approach to be investigated.

WP 3: Implementation of realistic test cases (reference trajectories)

The control structure should be tested in an environment as close to real conditions as possible. Typical departure routes e.g. from Munich airport are at the hand of the applicants and need to be translated to the exact reference trajectories (positions, velocity, course angle, etc.). Using these reference trajectories, solely the baseline control structure from WP 1 without adaptation is tested and benchmarked by appropriate metrics (e.g. maximum tracking error).

WP 4: Development of optimal and suboptimal testing procedures

For flight control law testing with respect to continuous disturbances an appropriate op-
Optimal control problem formulation will be used. This infinite dimensional problem is solved via direct methods, which translate the control problem to a large scale but sparse nonlinear optimization problem. In order to make this testing procedure applicable to the trajectory control law, it needs to handle very large state and control spaces. Thus, the efficiency of the transcription and the sparse evaluation algorithms is of paramount importance. Moreover, due to the local nature of large scale sparse nonlinear programming solvers, initial guess generation plays a major role for the quality of the solution. The investigation of homotopy methods for this specific problem setup e.g. based on gradually increasing the dynamic model complexity [28] will be part of this work package. Furthermore, “sharp” nonlinearities such as limiters need to be addressed carefully. One remedy to be investigated are appropriate relaxations, which can be included in the homotopy procedure. Moreover, time delays play an important role when testing control laws and should be taken into account e.g. via the “method of steps” [17].

WP 5: Preliminary testing using simplified models
In the current project, the incremental introduction of the differential game approach in the aircraft control architecture has shown to be very effective and allowed for detailed testing and preliminary performance evaluation. For this reason we would like to employ this procedure also for the approach under investigation. Therefore the test cases should build up regarding the nonlinearity of the model and the degrees of freedom (longitudinal, lateral). For distinct phases of flight, linearized models around steady state conditions such as horizontal steady state flight in cruise or around the glide slope in the landing phase provide a good starting point for preliminary testing of the control structure. Note that for the mentioned flight conditions, the longitudinal and lateral motion are dynamically decoupled, which allows for their initially separate treatment. Building up on this, the test cases can be extended by using first only the vertical nonlinear motion before extending it by the lateral nonlinear motion.

WP 6: Integration and final testing in the flight simulator
For the integration the interface of the flight simulator at FSD will be used. Final testing will be performed using the reference trajectories from WP 2 and the methods developed in WP 4 under realistic wind conditions and including sensor measurement errors.

WP 7: Documentation
Intermediate results obtained in course of the proposal’s implementation should be published. The overall work done and the results achieved need to be prepared for and documented in the final report.

2.4 Data handling
All data acquired during the project will be securely stored and kept available for further use on Server Systems provided by the Leibnitz Rechenzentrum servicing IT infrastructure for the Universities in Munich and other research facilities.
2.5 Other information

The applicants are planning publication of a book for the *SpringerBriefs in Control, Automation and Robotics*.

2.6 Descriptions of proposed investigations involving experiments on humans, human materials or animals

There will not be any experiments on humans, human materials or animals.

2.7 Information on scientific and financial involvement of international cooperation partners

There will not be any project related cooperation with international partners.
3 Bibliography

Unpublished project related publications


Other publications


