Problems Based on Lance Littlejohn's AbiTUMath Lecture

1. Prove that no odd prime number can be congruent to 2 modulo 4.
2. Prove that no prime number p that is congruent to 3 modulo 4 can be written as the sum of two squares.
3. Prove the following properties concerning continuants; here $q_i \in \mathbb{N}$.
   (a) $[q_1, q_2, \ldots, q_n] = q_1[q_2, q_3, \ldots, q_n] + [q_3, q_4, \ldots, q_n]$;
   (b) $[q_1, q_2, \ldots, q_n] = [q_n, q_{n-1}, \ldots, q_1]$;
   (c) $[q_2, q_3, \ldots, q_n] < [q_1, q_2, \ldots, q_n]$;
   (d) $\gcd([q_2, q_3, \ldots, q_n], [q_1, q_2, \ldots, q_n]) = 1$;
   (e) $[q_1, q_2, \ldots, q_{s-1}, q_s, q_{s+1}, q_{s+2}, \ldots q_n] =$
   $[q_1, q_2, \ldots, q_s][q_{s+1}, q_{s+2}, \ldots, q_n] + [q_1, q_2, \ldots, q_{s-1}][q_{s+2}, q_{s+3}, \ldots, q_n]$.
4. Verify in the statement of the Euclidean Algorithm that the integers r, s are given by
   $$r = [q_1, q_2, \ldots, q_n]$$
   and
   $$s = [q_2, q_3, \ldots, q_n].$$
5. Find the Smith numbers λ associated with the prime numbers $p = 29, 229, 317, 457, 521$. Express each of these primes as the sum of two squares.
6. Develop a computer program to generate the Smith numbers for *any* prime number p that is congruent to 1 modulo 4.