INTERNATIONAL CONFERENCE

PROGRESS ON DIFFERENCE EQUATIONS 2007

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Andreas Suhrer

Josef Suhrer

Laufen Colloquium on Science

PROGRESS ON DIFFERENCE EQUATIONS 2007

WELCOME REMARKS BY

THE PRESIDENT OF ISDE

Welcome Remarks by the President of the International Society of Difference Equations Professor Dr. Saber Elaydi

His Excellency the Mayor of Laufen,

His Excellency the President of Berchtesgaden district,

Dear Colleagues,

On behalf of the International Society of Difference Equations, I would like to welcome you all at this conference and in the beautiful town of Laufen. This is the second conference in the new series of meetings, titled "Progress on Difference Equations", and comes on the heels of the first conference in Homburg/Saar, Germany.

This meeting constitutes also an important development for the International Society of Difference Equations (ISDE). For the first time, a European town, namely the town of Laufen/Salzach, serves as an official organizer of a conference on difference equations.

Let me express my sincere gratitude and appreciation to the Mayor of Laufen, to the President of Berchtesgaden district and to the various sponsors: They all have demonstrated that the public awareness of advances in the sciences is as lively as ever.

It is noteworthy to mention that the focus on applications in mathematical biology fits very well with the scope of the hosting academy ANL (Akademie für Naturschutz und Landschaftspflege).

The research area of difference equations is relatively young. But the recent successes of various difference equations meetings in Augsburg 2001, Munich 2005, Homburg 2006 and Laufen 2007 have demonstrated that the subject of difference equations will continue to rise in its status and usage in almost all endeavors in life. Difference equations are now recognized as preferred models in natural sciences, engineering, computer science, social science, economics, etc.

Last but not least:

My sincere thanks go to our colleagues Priv.-Doz.Dr.Andreas Ruffing, OStD Josef Suhrer and Mr. Andreas Suhrer: They have done a marvelous job and have greatly contributed to ISDE by hosting this meeting. They have developed an excellent scientific program and a splendid social program.

I wish you all success in the conference and a joyful stay in Laufen.

Laufen Colloquium on Science

PROGRESS ON DIFFERENCE EQUATIONS 2007

SPEAKERS AND PARTICIPANTS

João Ferreira Alves, Lisboa, Portugal

Werner Balser, Ulm, Germany

Leonid Berezansky, Beer-Sheva, Israel

Joël Blot, Paris, France

Andrea Bruder, Waco, USA

Susanne Brunner, Munich, Germany

Alberto Cabada, Santiago de Compostela, Spain

Ricardo Coutinho, Lisboa, Portugal

Jim Cushing, Tucson, USA

Josef Diblík, Brno, Czech Republic

Saber N. Elaydi, San Antonio, USA

Michaela Ensinger, Munich, Germany

Thomas Ernst, Uppsala, Sweden

Stefan Gerhold, Vienna, Austria

Malgorzata Guzowska, Szczecin, Poland

István Győri, Veszprém, Hungary

Rafael Hernández Heredero, Madrid, Spain

Stefan Hilger, Eichstätt, Germany

Ale Jan Homburg, Amsterdam, The Netherlands

Klara R. Janglajew, Bialystok, Poland

Senada Kalabušić, Sarajevo, Bosnia and Herzegowina

Ulrich Krause, Bremen, Germany

Jan Lorenz, Bremen, Germany

Harald Markum, Vienna, Austria

Maria Meiler, Munich, Germany

Małgorzata Migda, Poznań, Poland

Henrique Oliveira, Lisboa, Portugal

P J S Pereira, Lisboa, Portugal

Miguel Piñar, Granada, Spain

Alberto A. Pinto, Porto, Portugal

Mihály Pituk, Veszprém, Hungary

Andrejs Reinfelds, Rīga, Latvia

J. Leonel Rocha, Lisboa, Portugal

Andreas Ruffing, Munich, Germany

Robert J. Sacker, Los Angeles, USA

Ewa L. Schmeidel, Poznań, Poland

Stefan Siegmund, Frankfurt, Germany

Luis Silva, Évora, Portugal

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Pavla Šnyrychová, Olomouc–Hejčín, Czech Republic

Stevo Stević, Beograd, Serbia

Andreas Suhrer, Munich, Germany

Imme van den Berg, Évora, Portugal

Andre Vanderbauwhede, Ghent, Belgium

Laufen Colloquium on Science

PROGRESS ON DIFFERENCE EQUATIONS 2007

AGENDA OF THE CONFERENCE

SUNDAY 1st April 2007 — Convention Center

OPENING OF THE CONFERENCE — **10:00–10:15**

S. Elaydi *(San Antonio)*
The Stochastic Beverton-Holt Equation with Survival Rates
And Almost Periodicity in the Mean — **10:15–11:15**

BREAK — **11:15–11:30**

J. Blot *(Paris)*
Discrete-Time Pontryagin Principles with Constraints — **11:30–12:00**

A. J. Homburg *(Amsterdam)*
Randomly Perturbed Diffeomorphisms — **12:00–12:30**

R. Coutinho *(Lisboa)*
Discontinuous Rotations — **12:30–13:00**

LUNCH BREAK — **13:00–14:30**

S. Gerhold *(Vienna)*
The Sign Structure of Linear Recurrence Sequences — **14:30–15:00**

J. Cushing *(Tucson)*
Difference Equations Models of Competition:
Multiple Attractors and Non-Equilibrium Coexistence — **15:00–16:00**

COFFEE BREAK — **16:00–16:30**

A. Reinfelds *(Rīga)*
Conjugacy of Difference Equations
in the Neighbourhood of Invariant Manifold — **16:30–17:00**

J. L. Rocha *(Lisboa)*
Ruelle Zeta Functions and Expanding
Maps on the Interval with Holes — **17:00–17:30**

M. Pituk *(Veszprém)*
Asymptotic Expansions for Difference Equations — **17:30–18:00**

RECEPTION BY THE LAUFEN MAYOR, MR. LUDWIG HERZOG

IN THE HISTORICAL OLD TOWN HALL OF LAUFEN — **19:00**

MONDAY 2nd April 2007 — Convention Center

A. Vanderbauwhede *(Ghent)* *Change of Stability and Branching of Periodic Orbits in Reversible Systems*	**09:00–09:30**
T. Ernst *(Uppsala)* *Operator Theory and q-Legendre, Cigler q-Laguerre, q-Jacobi Polynomials*	**09:30–10:00**
BREAK	**10:00–10:30**
R. Sacker *(Los Angeles)* *Dynamic Reduction, the Periodic Ricker Map and Genetically Altered Mosquitos*	**10:30–11:30**
I. Győri *(Veszprém)* *Asymptotic Representation of the Solutions of Linear Volterra Difference Equations*	**11:30–12:00**
LUNCH BREAK	**12:00–13:30**
U. Krause *(Bremen)* *Population Models with Non-Autonomous Dynamics*	**13:30–14:00**
J. Lorenz *(Bremen)* *Difference Equations in Continuous Opinion Dynamics under Bounded Confidence Interesting Types of Convergence*	**14:00–14:30**
COFFEE BREAK	**14:30–15:00**
DEPARTURE OF BUS FOR ROUNDTRIP	**15:30**
BOAT DEPARTURE TO ST. BARTHOLOMÄ	**18:00**

THEN RECEPTION BY THE DISTRICT'S PRESIDENT
OF BERCHTESGADEN COUNTY, MR. GEORG GRABNER
AT THE PICTURESQUE SITE OF ST. BARTHOLOMÄ

TUESDAY 3rd April 2007 — Convention Center

S. Hilger *(Eichstätt)*		
Deformation of the Heisenberg Commutator Relation	**09:00–09:30**	
A. Suhrer / A. Ruffing *(München)*		
Oscillation Behavior of Some Schrödinger Difference Equations	**09:30–10:00**	
BREAK	**10:00–10:30**	
K. Janglajew *(Bialystok)*		
On the Factorization of (q,h)*-Difference Operators*	**10:30–11:00**	
H. Markum *(Vienna)*		
The Solution of the Schrödinger Equation as an Application of Difference Equations	**11:00–11:30**	
R. Heredero *(Madrid)*		
On the Bäcklund Transformation Between the Complex Toda and Volterra Lattices	**11:30–12:00**	
I. van den Berg *(Évora)*		
A Theorem of De Moivre-Laplace of All Orders Obtained By Ordinary and Partial Difference Equations	**12:00–12:30**	
LUNCH BREAK	**12:30–13:30**	
M. Piñar *(Granada)*		
A Generating Function for Meixner–Sobolev Orthogonal Polynomials	**13:30–14:00**	
S. Stević *(Beograd)*		
Boundedness Character of some Classes of Nonlinear Difference Equations	**14:00–14:30**	
DEPARTURE OF BUS TO SALZBURG	**15:00**	
GUIDED TOUR THROUGH SALZBURG	**15:45–19:00**	
MOZART DINNER CONCERT AT RESTAURANT PETERSKELLER IN SALZBURG	**19:30**	

W. Balser *(Ulm)*
Formal Power Series Solutions for Systems of Linear Difference Equations **09:00–09:30**

S. Siegmund *(Frankfurt)*
Discrete versus Continuous and Asymptotic versus Transient Dynamics **09:30–10:00**

L. Berezansky *(Beer-Sheva)*
Nonoscillation and Stability for Linear Difference Equations with Several Delays **10:00–10:30**

BREAK **10:30–11:00**

A. Cabada *(Santiago de Compostela)*
First Order Difference Functional Equations with Nonlinear Functional Conditions **11:00–11:30**

A. Pinto *(Porto)*
Geometric Measures for Hyperbolic Sets on Surfaces **11:30–12:00**

J. Diblík *(Brno)*
Bounded Solutions of Nonlinear Discrete Equations **12:00–12:30**

P. Šnyrychová *(Olomouc-Hejčín)*
Chaos for Multi-Valued Mappings **12:30–13:00**

LUNCH BREAK **13:00–15:00**

M. Guzowska *(Szczecin)*
Stability of an Equilibrium Point in Goodwin's Growth Cycle Model **15:00–15:30**

S. Kalabušić *(Sarajevo)*
Non-hyperbolic Dynamics of Monotone Discrete Dynamical Systems **15:30–16:00**

J. Alves *(Lisboa)*
On the Set of Periods of a
Periodic Nonautonomous Difference Equation **16:00–16:30**

BREAK **16:30–17:00**

M. Migda *(Poznań)*
Asymptotically Polynomial Solutions
of Higher Order Difference Equations **17:00–17:30**

P. J. S. Pereira *(Lisboa)*
Orientational Director Effects in the
Continuum Theory of Nematic Liquid Crystals **17:30–18:00**

E. Schmeidel *(Poznań)*
Asymptotically Periodic Solutions
of Some Linear Difference Equations **18:00–18:30**

L. Silva *(Évora)*
Universal Convergence Rates
for Stunted Sawtooth Lorenz Maps **18:30–19:00**

DINNER **19:00–20:30**

GUIDED LANTERN WALKING WITH
A HISTORICAL NIGHT CUSTODIAN THROUGH LAUFEN **20:30**

Laufen Colloquium on Science

PROGRESS ON DIFFERENCE EQUATIONS 2007

ABSTRACTS AND ADDRESSES

On the Set of Periods of a Periodic Nonautonomous Difference Equation

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We consider the nonautonomous difference equation

$$x_{n+1} = f_n(x_n), \qquad (1)$$

where $\{f_n : [a,b] \to [a,b]\}_{n\geq 0}$ is a sequence of continuous interval maps. As usual we say that a positive integer, q, is a period of (1) if there exists a q-periodic solution of (1). Recall that the nonautonomous equation (1) is called p-periodic if $p \geq 2$ is the smallest positive integer satisfying $f_{n+p} = f_n$ for all $n \geq 0$.
This talk concerns to the set of periods of (1) when this equation is p-periodic. Our main theorem establishes that, if the equation is p-periodic, and the sets

$$\{x \in [a,b] : f_i(x) = f_j(x)\}$$

are finite for all $i \neq j$, then it is possible to compute explicitly all periods of the equation which are not a multiple of p.
Combining this result with a recent extension of Sharkovsky's theorem for periodic difference equations [1], we are able to describe the set of periods of the equation in many interesting situations.

References and Literature for Further Reading

[1] AlSharawi, Ziyad; Angelos, James; Elaydi, Saber; Rakesh, Leela. An extension of Sharkovsky's theorem to periodic difference equations, *J. Math. Anal. Appl.* 316 (2006), no. 1, 128–141.

[2] Artin, M.; Mazur,B., On periodic points, *Ann. of Math.* (2) 81 (1965), 82–99.

[3] Sharkovsky, A.N., Coexistence of cycles of a continuous transformation of a line into itself, *Ukrain. Mat. Zh.* 16 (1964) 61–71 (in Russian).

Formal Power Series Solutions for Systems of Linear Difference Equations

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A system of linear difference equations, whose coefficient matrix is meromorphic near the point infinity, may have power series solutions (in z^{-1}) that diverge for every value of z. We shall investigate whether these power series are multisummable in the sense of *Jean Ecalle*.

Nonoscillation and Stability for Linear Difference Equations with Several Delays

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We study a connection between the existence of positive solutions and exponential stability for a scalar linear difference equation with several delays:

$$x(n+1) - x(n) = -\sum_{l=1}^{m} a_l(n)x(h_l(n)), \;\; h_l(n) \le n, \;\; n > n_0.$$

Nonoscillation criteria, comparison theorems and some explicit nonoscillation and stability results are presented. Some known nonoscillation tests for equations with constant delays and with one variable delay are obtained as special cases.

References and Literature for Further Reading

[1] L. Berezansky, E. Braverman, On existence of positive solutions for difference equations with several delays, *Advanced in Dynamical Systems and Applications* 1 (2006) 29-47.

Discrete-Time Pontryagin Principles with Constraints

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We establish Pontryagin Maximum Principles for discrete-time infinite-horizon Optimal Control problems under constraints on the state and on the control. To prove these results we use a reduction to the finite horizon and we simultaneously work on the adjoint difference equation and on the pointwise maximization of the pre-Hamiltonian. The macroeconomic optimal growth theory is an example of scientific field which uses such Optimal Control problems.

References and Literature for Further Reading

[1] J. Blot, H. Chebbi, Discrete Time Pontryagin Principles with Infinite Horizon, *Journal of Mathematical Analysis and Applications* 246 (2000) 265-279.

[2] J. Blot, B. Crettez, The smoothness of optimal paths, *Decision in Economics and Finance* 27 (2004) 1-34.

[3] P. Michel, Some clarifications on the transversality condition, *Econometrica* 58 (1990) 705-723.

[4] N. L. Stockey, R. E. Lucas and E. C. Prescott, Recursive Methods in Economic Dynamics, Harvard University Press, Cambridge, Massachusetts, 1989.

First Order Difference Functional Equations with Nonlinear Functional Conditions

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It is studied a first order functional difference equation coupled with nonlinear functional boundary value conditions. Such boundary conditions include, among others, initial, periodic, antiperiodic and multipoint boundary value conditions, as a particular cases. By supposing the existence of a pair of well ordered lower and upper solutions, it is proved that the considered problem has at least one solution. Moreover, by assuming some suitable monotonicity properties on the boundary data, it is deduced that the studied problem has extremal solutions. For this situation, a monotone iterative technique is developed to approach these solutions.

References and Literature for Further Reading

[1] F. Atici, A. Cabada and J. B. Ferreiro, Existence and comparison results for first order periodic implicit difference equations with maxima, *Journal of Difference Equations and Applications,* 8 (2002), 4, 357-369.

[2] F. Atici, A. Cabada and J. B. Ferreiro, First order difference equations with maxima and nonlinear functional boundary value conditions. *Journal of Difference Equations and Applications,* 12 (2006), 6, 565-576.

[3] A. Cabada, J. B. Ferreiro and E. Liz, Comparison results for first order difference equations with maxima, *Proceedings of the XVII Congress of Differential Equations and Applications/VII Congress of Applied Mathematics*, (In Spanish), (2001).

[4] D. Franco, D. O'Regan and J. Perán, Upper and lower solution theory for first and second order difference equations. *Dynamics Systems & Applications*, 13 (2004) 2, 273–282.

Discontinuous Rotations

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We review the dynamics of the family of applications $x' = (ax + c)\ mod\ 1$, where a belongs to $(0, 1)$. This dynamical system has an attractor that is or a periodic orbit or a Cantor set where the evolution is semi-conjugated to an irrational rotation. Using explicit symbolic dynamics we find a unique formalism that describe (for all parameters a and c) all orbits independently of the rationality of the rotation number. The dependence of this number with each parameter is proven to be a Cantor staircase (explicit formulas are given). Finally we obtain an efficient and simple algorithm that gives the continued fraction expansion of the rotation number.

References and Literature for Further Reading

[1] Y. Bugeaud, J-P. Conze, Calcul de la dynamique de transformations linaires contractantes mod 1 et arbre de Farey, *Acta Arith.* 88 (1999) no. 3, 201-218.

[2] R. Coutinho, Dinâmica Simbólica Linear, (PhD Thesis), Universidade Técnica de Lisboa - IST (1999).

[3] P. Veerman, Symbolic Dynamics Of Order-preserving Orbits, *Physica D* 29 (1987) 191-201.

Difference Equation Models of Competition: Multiple Attractors and Non-Equilibrium Coexistence

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Difference equations have historically played (and continue to play) a significant role as models for the dynamics of biological populations. An example comes from competition theory in which difference equations (in addition to the familiar ordinary differential equations named after Lotka and Volterra) were used to establish the fundamental Competitive Exclusion Principle. During the 1950-1960's this Principle was controversial and, in order to help clarify and focus the issues involved, several researchers conducted controlled laboratory experiments to test it (specifically, to test the assertion that two species, in order to coexist, must find a way to decrease the competitive intensity between them). While G. F. Gause utilized the Lotka-Volterra differential equations to guide his experiments, T. Park and his collaborators used a difference equation analog of these equations [1]. Although these experimental results are now considered as definitive demonstrations of the Competitive Exclusion Principle (and to this day are widely cited in basic ecology texts), one of Park's results was anomalous in that the two species in this exceptional case coexisted when the theory predicted one should have gone extinct. Park and his collaborators (who included the well-known P. H. Leslie) did not ignore this "coexistence case", but studied it in two follow-up papers in which they ultimately decided that they had no explanation for its occurrence [2,3]. A possible explanation is offered in a recent paper by Edmunds et al. [4] in which a more complicated competition model is utilized. That model is based on a well validated, difference equation model that has had significant success in describing and predicting the dynamics of the specific organism used by Park in his classic experiments (*Triolium castaneum*) [5,6]. The explanation given by Edmunds et al. [4], however, seemingly contradicts the classic Competitive Exclusion Principle by allowing coexistence under *increased* competitive intensity. The dynamic scenario involves an unusual multiple attractor configuration that includes both coexistence and exclusion attractors. My talk will summarize recent research on simpler (lower dimensional) models designed to determine what mechanisms (mathematical and biological) are responsible for such dynamic scenarios in competition models.

References and Literature for Further Reading

[1] P. H. Leslie and J. C. Gower, The properties of a stochastic model for two competing species, *Biometrika* 45 (1958), 316-330.

[2] T. Park, P. H. Leslie and D. B. Mertz, Genetic strains and competition in populations of *Tribolium*, *Physiological Zoology* 37 (1964), 97–162.

[3] P. H. Leslie, T. Park and D. M. Mertz, The effect of varying the initial numbers on the outcome of competition between two *Tribolium* species, *Journal of Animal Ecology* 37 (1968), 9–23.

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[6] J. M. Cushing, R. F. Costantino, B. Dennis, R. A. Desharnais and S. M. Henson, *Chaos in Ecology: Experimental Nonlinear Dynamics*, Academic Press, New York, 2002.

Bounded Solutions of Nonlinear Discrete Equations

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We study a problem concerning the compulsory behavior of the solutions of systems of discrete equations $u(k+1) = F(k, u(k))$, $k \in N(a) = \{a, a+1, a+2, \dots\}$, $a \in \mathbb{N}$, $\mathbb{N} = \{0, 1, \dots\}$ and $F : N(a) \times \mathbb{R}^n \to \mathbb{R}^n$. A general principle for the existence of at least one solution with graph staying for every $k \in \{a, a+1, a+2, \dots\}$ in a previously prescribed domain is formulated. Such solutions are defined by means of the corresponding initial data and their existence is proved by means of retract type approach. For the development of this approach a notion of egress type points lying on the defined boundary of a given domain and with respect to the system considered is utilized. Unlike previous investigations, the boundary can contain points which are not points of egress type, too. (This research was supported by the Council of Czech Government MSM 2622000 13 and MSM 0021630519.)

References and Literature for Further Reading

[1] J. Diblík, Anti-Lyapunov method for systems of discrete equations, *Nonlinear Anal.* **57** (2004), 1043-1057.

[2] J. Diblík, M. Migda, E. Schmeidel, Bounded solutions of nonlinear discrete equations, *Nonlinear Anal.* **65** (2006), 845-853.

[3] J. Diblík, M. Růžičková, B. Václavíková, A retract principle on a time scale, *Stud. Univ. Žilina Math. Ser.* **18** (2004), 37-44.

[4] J. Diblík, M. Růžičková, B. Václavíková, A retract principle on discrete time scales, *Opusc. Math.* **26** (2006), 445-455.

[5] J. Diblík, M. Růžičková, I. Růžičková, A general version of the retract method for discrete equations *Acta Math. Sin.* **23** (2007), 341-348.

The Stochastic Beverton-Holt Equation with Survival Rates And Almost Periodicity in the Mean

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The paper studies a Beverton-Holt difference equation, in which both the recruitment function and the survival rate vary randomly. It is then shown that there is a unique invariant density, which is asymptotically stable. Moreover, a basic theory for random mean almost periodic sequence on Z_+ is given. Then, some sufficient conditions for the existence of a mean almost periodic solution to the stochastic Beverton-Holt equation are given.

References and Literature for Further Reading

[1] T. Diagana, S. Elaydi, A-A. Yakubu, Population Models in Almost Periodic Environments. To Appear in *J. Difference Equations and Appl.* (2006).

[2] C. Haskell and R. Sacker, The Stochastic Beverton-Holt Equation and the M. Neubert Conjecture. *J. Dynam. Diffential Eq.* **17** (2005), no. 4, 825–844

Operator Theory and q-Legendre, Cigler q-Laguerre, q-Jacobi Polynomials.

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We develop an operational calculus in $(\mathbb{C}(q))[[x]]$, which includes the q-Gould-Hopper formula, 6 q-analogues of the Carlitz' expression for the q-Laguerre Rodriguez operator. The corresponding formulas for q-Jacobi-polynomials are found in the same way, but include fields of fractions for the multiplication operator. A q-analogue of Bailey is shown and orthogonality relations are found. The q-difference equation for q-Legendre polynomials is one example of spectral theory for second order q-difference equations. q-Schrödinger equations is another example with partial q-difference equations. The confusion of tongues between mathematicians and physicists with respect to Laguerre polynomials is considered.

References and Literature for Further Reading

[1] C.G.J. Jacobi, *Werke 6* Berlin (1891).

[2] L. Carlitz, A note on the Laguerre polynomials. *Michigan Math. J.* **7** (1960) 219–223.

[3] J. Cigler, Operatormethoden für q-Identitäten II, q-Laguerre-Polynome, *Monatshefte für Mathematik* **91** (1981) 105-117.

[4] W. N.Bailey, On the product of two Laguerre polynomials, *Quart. J. Math* **10** (1939) 60-66.

[5] E. Schrödinger, Quantisierung als Eigenwertproblem. III. *Annalen d. Physik* (4) **80** (1926) 437-490.

The Sign Structure of Linear Recurrence Sequences

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Not as easy as it may at first glance seem, the question whether a linear recurrence sequence, obeying a constant coefficient recurrence, is eventually positive poses some delicate number theoretic questions. The case of several dominating characteristic roots on the unit circle calls for Diophantine methods to investigate the sign structure. In concrete examples, computer algebra can be invoked to reveal patterns in the signs of the sequence elements.

References and Literature for Further Reading

[1] J. P. Bell and S. Gerhold, On the Positivity Set of a Linear Recurrence Sequence, *Israel J. Math.,* to appear.

[2] J. R. Burke and W. A. Webb, Asymptotic behavior of linear recurrences, *Fibonacci Quart.* 19 (1981) 318–321.

[3] S. Gerhold, Point Lattices and Oscillating Recurrence Sequences, *Journal of Difference Equations and Applications* 11 (2005) No. 6, 515–533.

[4] S. Gerhold and M. Kauers, A Procedure for Proving Special Function Inequalities Involving a Discrete Parameter, *Proceedings of ISSAC '05, M. Kauers (ed.), ACM Press* (2005) 156–162.

Stability of an Equilibrium Point in Goodwin's Growth Cycle Model

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In theory of economics, most models describing economic growth use differential equations.
However, when trying to use them by econometricians, many questions arise. First, economic data (especially macroeconomic) are discrete, what forces use of difference equations. Second, transformation from continuous to discrete form of a model is still controversial.
The essence of above-mentioned problems and proposal of solving them will be presented on the basis of Goodwin's Growth Cycle Model.

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Asymptotic Representation of the Solutions of Linear Volterra Difference Equations

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A class of linear Volterra difference equations and inequalities is considered on finite dimensional space. It is proved that under appropriate assumptions the solutions are bounded even more they converge to limits as $n \longrightarrow +\infty$. The limits are given through the inhomogenity, the kernel and the initial values. As corollary of our results we also present some stability theorems. It is important to note that our results are applicable when the investigated equation does not have real characteristic value.

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On the Bäcklund Transformation Between the Complex Toda and Volterra Lattices

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We give sufficient conditions for the existence of the Bäcklund transformation between the semiinfinite Toda and Volterra lattices, in the complex case. For that purpose, we use as an intermediate technical tool a sequence of polynomials associated to the Jacobi matrix that appears in the Lax pair of the Toda lattice. The existence of the Bäcklund transformation is related to the set of zeroes of these polynomials, which in turn can be characterized by a spectral result on the associated Jacobi matrix.

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Deformation of the Heisenberg Commutator Relation

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The Heisenberg (or canonical) commutator relations (CCR)

$$DX - XD = I$$

and the corresponding Weyl algebra are realized on $L^2(\mathbb{R})$ by the classical (unbounded) operators $Xf(x) = x \cdot f(x)$ and $Df(x) = f'(x)$. There is a well known deformation of the CCR given by

$$D_q X - qXD_q = I$$

realized by the q difference operator $D_q f(x) = \frac{f(qx)-f(x)}{qx-x}$. The central difference operator $D_h f(x) = \frac{f(x+h)-f(x-h)}{2h}$ leads to another deformation.

In the talk we will show that a certain generalized ladder formalism provides the right algebraic framework for these deformations.

Randomly Perturbed Diffeomorphisms

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Using randomly perturbed circle diffeomorphisms as guide, I will review the dynamics and bifurcations of diffeomorphisms with bounded noise. Of particular interest are bifurcations where the support of a stationary measure explodes. I will explain this scenario and discuss quantitative characteristics. This is joint work with Hicham Zmarrou.

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On the Factorization of (q, h)-Difference Operators

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In the present lecture we develop a general result concerning the factorization of linear difference and differential operators [3 and 4].
A simple, direct proof that the linear (q, h)-difference operator of order mth

$$L^{(q,h)}y = \partial^m_{(q,h)}y + \sum_{k=0}^{m-1} a_k(x)\partial^k_{(q,h)}y,$$

where $a_0 \neq 0$,$\partial_{(q,h)}$ is the (q, h)-derivative [2]

$$\partial_{(q,h)}y(x) := \frac{y(x) - y(qx + h)}{(1 - q)x - h},\ 0 < q < 1,\ \ h > 0.$$

admits a factorization into first order linear (q, h)-operators will be given.
We also present the limiting cases when $q \to 1$,$h \to 0$ and some applications.

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Non-hyperbolic Dynamics of Monotone Discrete Dynamical Systems

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We present the attractivity results for monotone discrete dynamical systems of the form

$$\begin{aligned} x_{n+1} &= f(x_n, y_n) \\ y_{n+1} &= g(x_n, y_n), \quad n = 0, 1, 2, \ldots \end{aligned}$$

where f and g are continuous functions and $f(x, y)$ is non-decreasing in x and non-increasing in y and $g(x, y)$ is non-increasing in x and non-decreasing in y in some domain $\mathcal{A}$ in a non-hyperbolic case. We assume that the system has an infinite number of equilibrium points which belong to a linearly ordered set with respect to the south=east ordering. We give some examples that illustrate our results .

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Population Models with Non-Autonomous Dynamics

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It is quite natural that the parameters in difference equations which model biological populations are time–dependent, in particular periodic due to different seasons. In the talk, a general theorem is presented which yields for periodic driven systems the global convergence to stable cycles. This theorem applies in particular to non-autonomous Beverton-Holt equations as well as to more nasty Riccati equations. These equations are steered by so-called cave maps which will be defined and studied in detail.

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Difference Equations in Continuous Opinion Dynamics under Bounded Confidence Interesting Types of Convergence

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We study state- and time-discrete inhomogeneous Markov chains which are derived with the heuristics of opinion dynamics under bounded Confidence as proposed in the models of Hegselmann-Krause and Deffuant-Weisbuch. The inhomogeneous Markov chain gives the represents the dynamics of the density of agents in the opinion space. The derived difference equation is in analogy to a master equation that represents for each opinion the fraction of agents that join the opinion and the fraction that leaves this opinion. So, the set of fixed points for both models can be derived.

The systems shows an interesting convergence behavior which leads to clusters in the opinion space. The system always converges to fixed points but the fixed points are not isolated in the state space but. Further on, the Markov chain is not weakly ergodic and the interrelation of the limit state and the initial state is not fully understood. Thus, simulation results will be presented to showing bifurcations of the opinion clusters regarding the bound of confidence.

For references please turn page!

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The Solution of the Schrödinger Equation as an Application of Difference Equations

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This talk serves as a description of how solutions of the Schrödinger equation can be characterized by difference equations.

Asymptotically Polynomial Solutions of Higher Order Difference Equations

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We consider the asymptotically polynomial solutions of difference equations of the form

$$\Delta^m x_n + f(n, x_n) = 0. \tag{2}$$

We present sufficient conditions under which every solution of equation (1) is asymptotically polynomial of degree $\leq m$ (the sequence (x_n) is called asymptotically polynomial of degree k if there exists a polynomial $P(n)$ of degree k such that $x_n = P(n) + o(1)$). We give also sufficient conditions under which for every polynomial of degree $\leq m$ there exists a solution (x_n) a of equation (1) such that $x_n = P(n) + o(1)$.
Moreover, we will consider the difference equations of neutral type

$$\Delta^m (x_n - p_n x_{n-\tau}) + f(n, x_n) = 0 \tag{3}$$

and give sufficient conditions under which every nonoscillatory solution of equation (2) is asymptotically polynomial of degree $\leq m$.
Some related problems will also be discussed.

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Orientational Director Effects in the Continuum Theory of Nematic Liquid Crystals

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Orientational director effects in nematic liquid crystals driven by external forces are investigated. The continuum theory of nematic liquid crystals leads to a system of non-linear differential equations to describe the spatial orientation of the director field. Analytical methods are used to solve these non-linear differential equations obtained from a modified and more convenient form of the Ericksen-Leslie partial differential equations. Some hydrodynamic limits are considered, in particular, flow regimes with small Ericksen number. Analytical solutions are presented for some boundary conditions and some physical interpretation is given.

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A Generating Function for Meixner–Sobolev Orthogonal Polynomials

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Let $\{S_n(x)\}_{n\geq 0}$ denote the sequence of polynomials orthogonal with respect to the Sobolev inner product

$$(f,g)_S = \sum_{k=0}^{+\infty} f(k)g(k)\frac{c^k(\beta)_k}{k!} + \lambda \sum_{k=0}^{+\infty} \Delta f(k)\Delta g(k)\frac{c^k(\beta)_k}{k!},$$

where $\beta > 0, 0 < c < 1, \lambda \geq 0$ and the leading coefficient of the $S_n(x)$ is equal to the leading coefficient of the Meixner polynomial $m_n(x;\beta,c)$. In this work, a generating function for the Meixner–Sobolev polynomials is obtained.

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Geometric Measures for Hyperbolic Sets on Surfaces

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We present a moduli space for all hyperbolic basic sets of diffeomorphisms on surfaces that have an invariant measure that is absolutely continuous with respect to Hausdorff measure. To do this we introduce two new invariants: the measure solenoid function and the cocycle-gap pair. We extend the eigenvalue formula of A. N. Livšic and Ja. G. Sinai for Anosov diffeomorphisms which preserve an absolutely continuous measure to hyperbolic basic sets on surfaces which possess an invariant measure absolutely continuous with respect to Hausdorff measure. We characterise the Lipschitz conjugacy classes of such hyperbolic systems in a number of ways, for example, in terms of eigenvalues of periodic points and Gibbs measures.

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Asymptotic Expansions for Difference Equations

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We give an asymptotic description of the solutions of higher order nonlinear difference equations in a neighborhood of an equilibrium.

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Conjugacy of Difference Equations in the Neighbourhood of Invariant Manifold

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In Banach space $\mathbf{X} \times \mathbf{S}$ the system of difference equations

$$\begin{aligned} x(t+1) &= A(s(t))x(t) + f(s(t), x(t)), \\ s(t+1) &= S(s(t), x(t)) \end{aligned}$$

is considered. Sufficient conditions under which the the difference system is simpler than the given one in terms of decoupling and linearization are obtained. This result allows one to replace the given system by a much simpler one. Relevant results concerning partial decoupling and simplifying of the noninvertible difference equations are given also.

Ruelle Zeta Functions and Expanding Maps of the Interval with Holes

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In this talk we discuss a class of open chaotic dynamical systems, from the point of view of dynamical zeta functions. Working with expanding maps of the interval with holes and dynamical zeta functions, we show how to compute explicitly important topological and metrical invariants of the system.

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Oscillation Behavior of
Some Schrödinger Difference Equations

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We present some examples for oscillation behavior in Schrödinger difference equations. The oscillation is investigated in context of the spectral properties of the operators under consideration.

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Dynamic Reduction, the Periodic Ricker Map and Genetically Altered Mosquitos

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Dynamic reduction is a problem dependent algorithm which allows one to solve a problem by reducing it to a sequence of simpler ones, e.g. consider the problem of finding a p-periodic solution to the p-periodic difference equation

$$x_{n+1} = f_n(x_n), \qquad f_{n+p} = f_n.$$

It may happen that the dependence of f_n on the state variable x decomposes into a convenient form

$$f_n(x) = F_n(x, g(x))$$

such that for all $v \in \mathcal{P}_p \subset \{p\text{-periodic sequences}\}$, the reduced equation

$$x_{n+1} = F_n(x_n, g(v_n))$$

has a unique p-periodic solution $v^* \in \mathcal{P}_p$. One then has an induced mapping

$$\mathcal{T} : \mathcal{P}_p \to \mathcal{P}_p, \qquad v^* = \mathcal{T}(v).$$

A **fixed point** of $\mathcal{T}$ yields a solution of $x_{n+1} = f_n(x_n)$. The procedure will be illustrated to obtain a globally asymptotically stable solution for a system of difference equations modeling the interaction of wild and genetically altered mosquitoes.

Asymptotically Periodic Solutions of Some Linear Difference Equations

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In this talk we set together various basic statements on the periodicity of the solutions of first order linear difference equations. Finally, some classes of a higher order linear difference equations are considered. Sufficient conditions for the existence of an asymptotically periodic solutions are given. The results are illustrate on many examples.

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Discrete vs. Continuous & Asymptotic vs. Transient Dynamics

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To study a real world dynamical process is it better to investigate the asymptotic behavior of a continuous time model (e.g. attractor of differential equation) or is it more profitable to analyze the discrete time process (difference equation) generated by either a numerically computed discretization or a physically observed data set on a "transient" finite time interval? We will not answer this question but present some typical tools for both approaches such as *time scales calculus*, the *Sacker Sell spectrum*, *Benfords law* and the *dynamic partition*.

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Universal Convergence Rates for Stunted Sawtooth Lorenz Maps

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We consider a family of piecewise monotonic one-dimensional maps with horizontal and vertical gaps. Using symbolic dynamics, we describe the structure of the bifurcation skeleton and explicitly calculate the convergence rates for sequences of parameters corresponding to homoclinic bifurcations, related with successive renormalizations. This rates are universal for this families of maps.

Chaos for Multi-Valued Mappings

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Since in the literature chaos is defined only for single-valued functions, our first aim is to suggest definitions of chaotic multi-valued mappings and their properties. The idea for definition of chaos is taken from the series of papers by different authors (see e.g. [3],[4],[5]). Although they all use the notion of "set-valued chaos", they only consider chaos for two single-valued functions; one on compact metric space X and the other on the hyperspace $\mathcal{K}(X)$ of all non-empty compact subsets of X. Edalat [6] replaced the hyperspace $\mathcal{K}(X)$ by the so-called upper hyperspace $\mathcal{U}(X)$ and described analogous relationships between (again single-valued) chaos in the original space X and this hyperspace.

Our goal is to describe the above situation when we consider a multi-valued mapping (on the original space X) instead of a single-valued one.

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Boundedness Character of some Classes of Nonlinear Difference Equations

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Recently there has been a great interest in studying the boundednes character of rational and nonlinear difference equations. We present here an account of results regarding the boundedness character, with a special attention to positive solutions of the following difference equation

$$x_n = A + \frac{x_{n-k}^p}{x_{n-m}^p}, \quad n \in \mathbf{N}_0,$$

where m and k are fixed natural numbers and $p \in (0, \infty)$.

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A Theorem of DeMoivre-Laplace of All Orders Obtained by Ordinary and Partial Difference Equations

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A well-known proof of the DeMoivre-Laplace central limit theorem establishes a difference equation for the binomial coefficients $\binom{n}{k}$, which after normalization and going to the limit changes into a differential equation for the Gaussian curve. It is not difficult to extend this approach to the whole Pascal triangle, and then to obtain through a partial difference equation or through two ordinary difference equations in the limit the two-dimensional Gaussian function $G(t,x) = \left(1/\sqrt{2\pi t}\right) \exp\left(-x^2/2t\right)$.
We use here the two ordinary difference equations and the partial difference equation to prove the convergence for all orders, in the sense that for all m the central difference quotients of order m of the binomial coefficients, after normalization, converge to the corresponding partial derivatives of order m of G.

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Change of Stability and Branching of Periodic Orbits in Reversible Systems

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In reversible systems symmetric periodic orbits typically appear in one-parameter families. In this talk we discuss the different possibilities for branching of other families from a given family when along the given family there is a "change of stability", in the sense that a pair of simple multipliers moves along the unit circle towards $+1$ and then splits off the unit circle along the real axis. Using the Poincaré map and Liapunov-Schmidt reduction this problem reduces to a 3-dimensional bifurcation problem in which the reversibility and possible other symmetries play an important role. We briefly discuss an application to a set of Discrete Non-Linear Schrödinger Equations coming from laser physics.

This is joint work with Eusebius Doedel (Montréal).