

The Southern Pine Beetle  
A Preliminary Time Scale Model

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*Dendroctonus frontalis*  
The Southern Pine Beetle



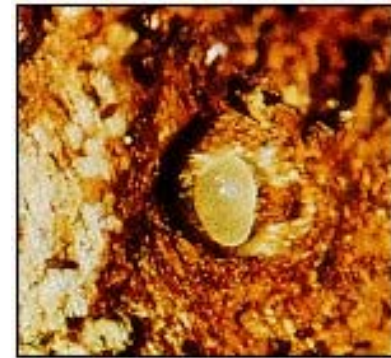
## The Southern Pine Beetle is:

- a species of bark beetle that lives in forests in the southern United States and Central America
- the most destructive forest insect pest in the southeastern U.S
- the cause of \$900 million of damage to pine forests from 1960 through 1990
- damaging not just through tree death but also through an associated “blue-stain” fungus that reduces the value of salvaged timber

Each attack involves one pair of beetles. When populations are large, thousands of beetles may invade individual trees. Each pair constructs a winding gallery in the inner bark and the female deposits pearly-white eggs in individual niches along the sides. The galleries, which frequently meet or cut across one another, girdle the tree. Blue-stain fungi, carried by the beetles, hasten the death of the tree by plugging the water-conducting tissues.

The egg hatches into a whitish, crescent-shaped larva with a glossy, reddish-brown head. When fully developed, the larva is about 3mm long. Newly hatched larvae mine in the soft inner bark; older larvae mine outward into the corky bark.

When fully grown, the larva changes to the resting stage or pupa, which is pure white and very soft.



A



B



C

When pupation is complete, the newly formed adults chew individual exit holes through the bark and take flight. Emerging beetles may invade green trees in the vicinity or fly considerable distances to begin new infestations.



Young Adult ready to emerge



UGA0745057

Actual size of adults



# Affected Forest



# Pitch Tube



## Blue-stain in log



## Larva and Pupa in outer bark



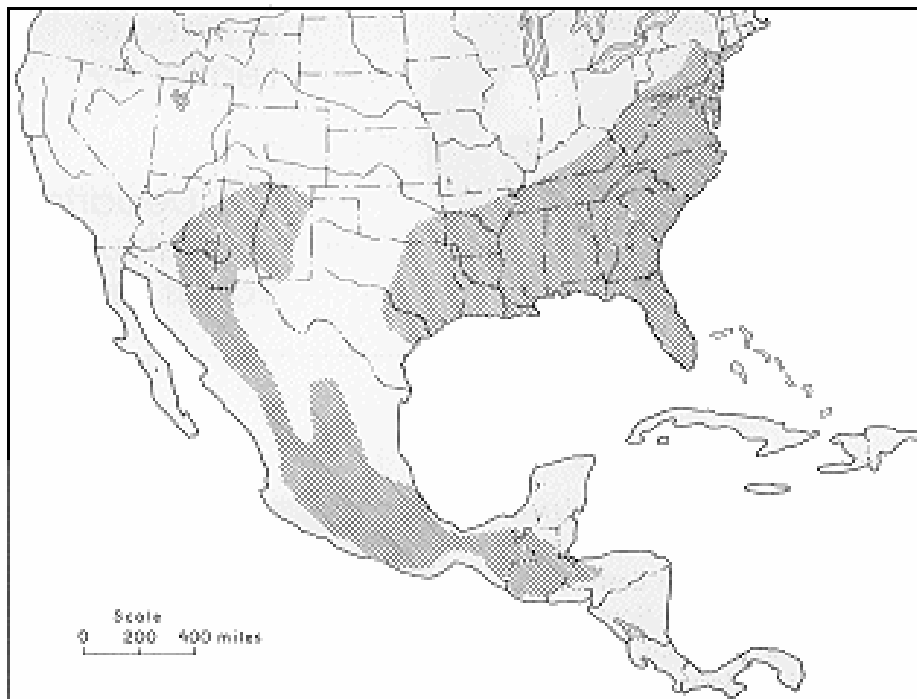
# Larvae tubes



# Popcorn resin tube



## Southern Pine Beetle Range



- Adults from over-wintering broods emerge and begin to attack uninfested trees in early spring-about the time dogwood trees flower.
- Depending upon latitude and elevation, there may be three to seven generations each year.
- Often, all life stages are present in different trees in the same infestation.
- During outbreaks, beetle activity peaks in early summer in the Gulf States and in late summer and early fall farther north.
- Under ideal conditions, the number of beetles may increase tenfold in a single generation and sparse populations may reach epidemic proportions within a summer.

The length of the different life cycles of the SPB depends largely on temperature.

| <i>Life Stage</i> | <i>Length of Life Stage</i>   |
|-------------------|---|
| Egg               | 11 days at 15° C, 3 days at 30° C,<br>up to 34 days at temperatures as low as 10° C |
| Larva             | 40 days at 15° C, 15 days at 25° C  |
| Pupa              | 17 days at 15° C, 5 days at 30° C   |
| Adult             | 14 days at 15° C, 6 days at 30° C   |
| <b>Total</b>      | 105 (low temperatures)  |
| <b>Days</b>       | 29 days (high temperatures)   |

## The Hollis Model (differential model)

One unit of time represents an average Southern Pine Beetle lifespan of 8 weeks, so there are 6.5 time units per year.

|     |  |
|-----|--|
| $p$ | scaled average density of living trees, assumed to be proportional to the average attackable surface area per acre |
| $u$ | density of trees that are capable of fending off an attack   |
| $w$ | density of trees that are vulnerable to SPB attack   |
| $b$ | scaled average pine beetle density   |

Assume that  $p$  satisfies a logistic growth equation when  $b=0$ .

Using data from the International Paper Company, the intrinsic per capita growth rate is approximately  $1/40$  per unit time.

Then  $p$  is scaled so that the stable equilibrium of this logistic model is at  $p=1$ . So, when  $b=0$ ,

$$p' = (1/40)p(1-p),$$

or

$$u' + w' = (1/40)(u+w)(1-(u+w)).$$

The surface area of the tree is assumed to become vulnerable to Southern Pine Beetle attack at a rate which is proportional to  $u$  and independent of  $b$ .

Thus

$$u' = (1/40)(u+w)(1-(u+w)) - au,$$

where  $a > 0$ , and if  $b = 0$ ,

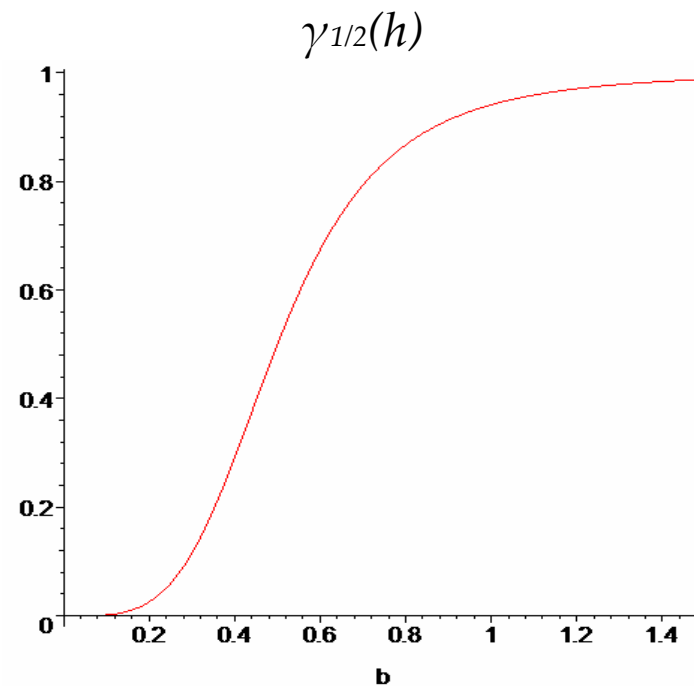
$$w' = au.$$

The range of the parameter  $a$  is roughly between 0.003 and 0.015.

Due to Southern Pine Beetle attack,  $w$  is assumed to decrease at a rate given by  $w\gamma_h(b)$  where the predatory functional response has the form

$$\gamma_h(b) = b^4/(h^4 + b^4).$$

Notice that  $\gamma_h(h) = 1/2$  and  $\gamma_h(b)$  is approximately 1 for “large” values of  $b$ .



The attack density is defined as

$$\theta_\varepsilon = b/(\varepsilon u + (1-\varepsilon)w),$$

where  $0 \leq \varepsilon \ll 1$ . The parameter  $\varepsilon$  is small because the SPBs are expected to have a strong preference for vulnerable trees.

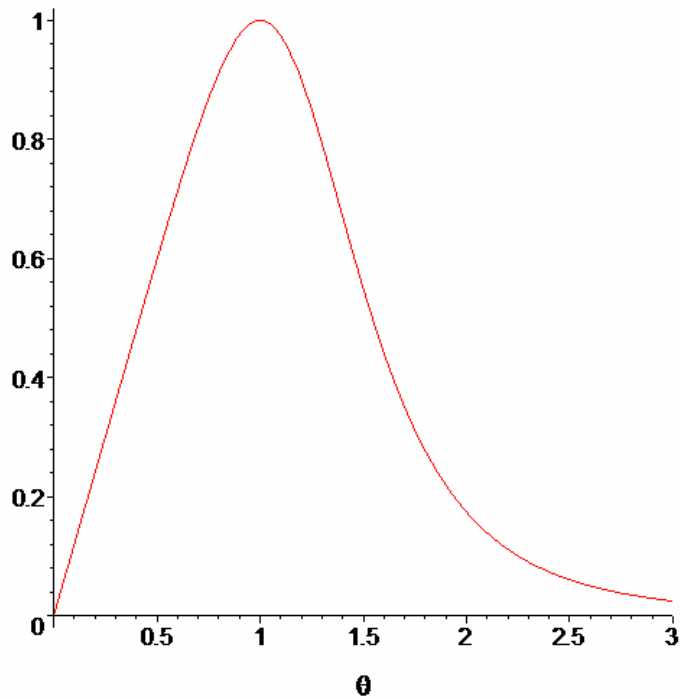
It is assumed that the per capita birth rate of the SPBs is given by  $\lambda w \sigma_n(\theta_\varepsilon)$  where  $\lambda > 0$  and

$$\sigma_n(\theta_\varepsilon) = (6\eta^5\theta)/(5\eta^6 + \theta^6) .$$

This function models the rate of emergence of adult SPBs per unit surface area as a function of the attack density.

The maximum value is attained at  $\sigma_\eta(\eta)=1$ .

The rapid decrease for values above the threshold parameter reflects the intraspecific scramble competition among Southern Pine Beetle larvae.



The threshold parameter  $\lambda$  reflects the intraspecific scramble competition among SPB larvae. It is related to the number of eggs laid per female SPB, which can be as large as 30. Since this model does not distinguish between male and female SPBs, it is reasonable to assume that  $\lambda$  falls between 5 and 15.

It is assumed that the SPB immigration rate is given by  $kp$ , where  $k > 0$ . It is expected that  $k$  will be relatively small.

These assumptions lead to the system of ordinary differential equations:

$$\begin{aligned} u' &= (1/40)(u+w)(1-(u+w)) - au \\ (1) \quad w' &= au - w \gamma_h(b) \\ b' &= b(\lambda w \sigma_n(\theta_\varepsilon) - 1) + k(u+w). \end{aligned}$$

Hollis provides numerical experiments to show that the  $b$  component of the system exhibits periodic “outbreaks” for a variety of sets of reasonable parameter values.

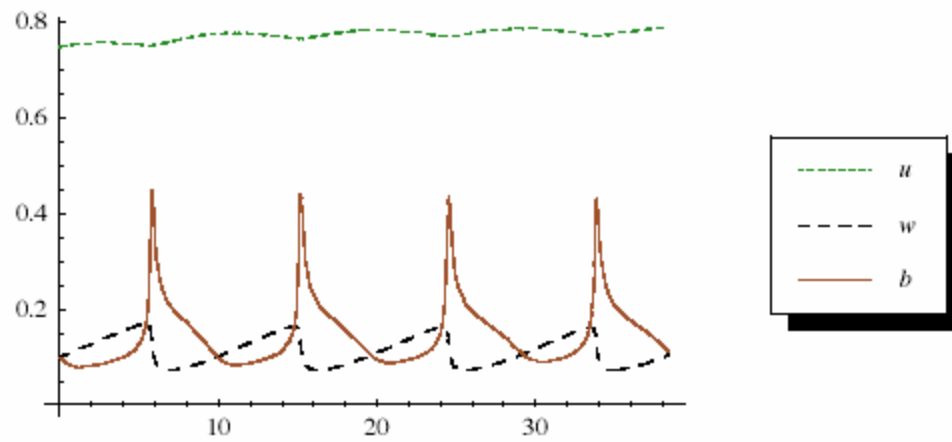


Fig. 4. A solution of (8) with  $\alpha = .003$ ,  $h = .5$ ,  $\lambda = 10$ ,  $\varepsilon = .05$ ,  $\eta = 2$ ,  $k = .057$ . (Horizontal scale in years.)

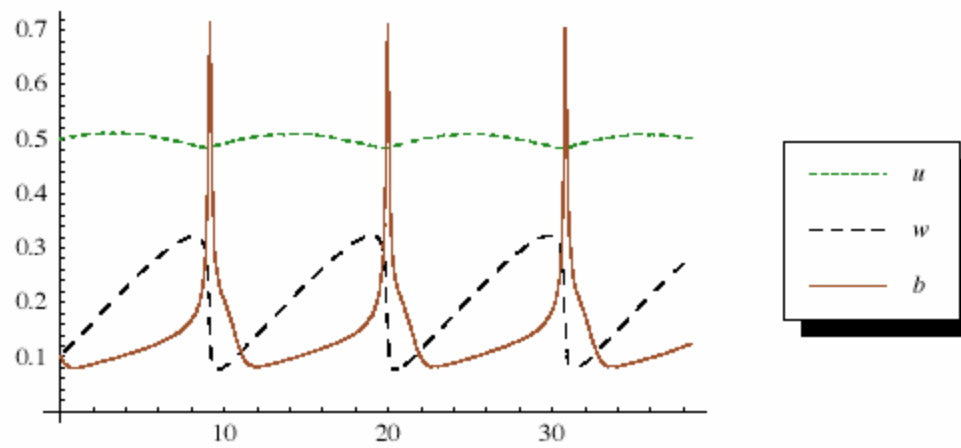


Fig. 5. A solution of (8) with  $\alpha = .01$ ,  $h = .5$ ,  $\lambda = 9$ ,  $\varepsilon = 0$ ,  $\eta = 3$ ,  $k = .09$ . (Horizontal scale in years.)

## Preliminary Time Scale Results

Consider:

$$p' = (1/40)p(1-p).$$

The equivalent time scale equation is:

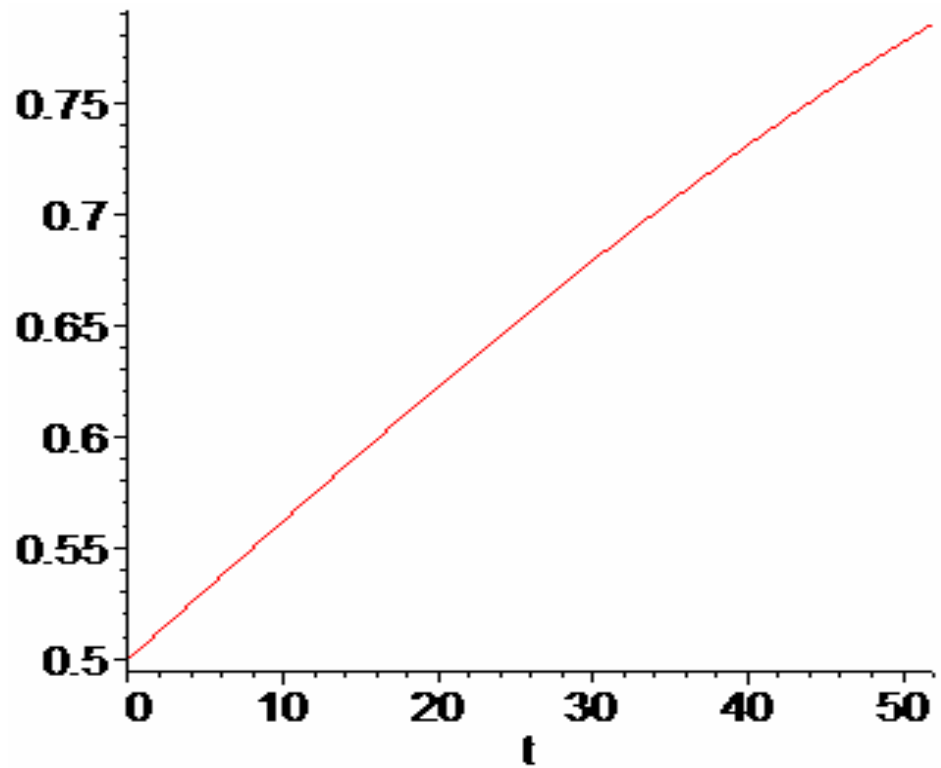
$$p^\Delta = (1/40)p(1-p).$$

We assume that the life span of the Southern Pine Beetle is not the average 8 weeks. Instead it varies from 4 weeks to 15 weeks. Thus a typical time scale (starting in March when the eggs hatch after winter) would be

$$\{0, 10, 17, 25, 31, 36, 40, 52, \dots\}$$

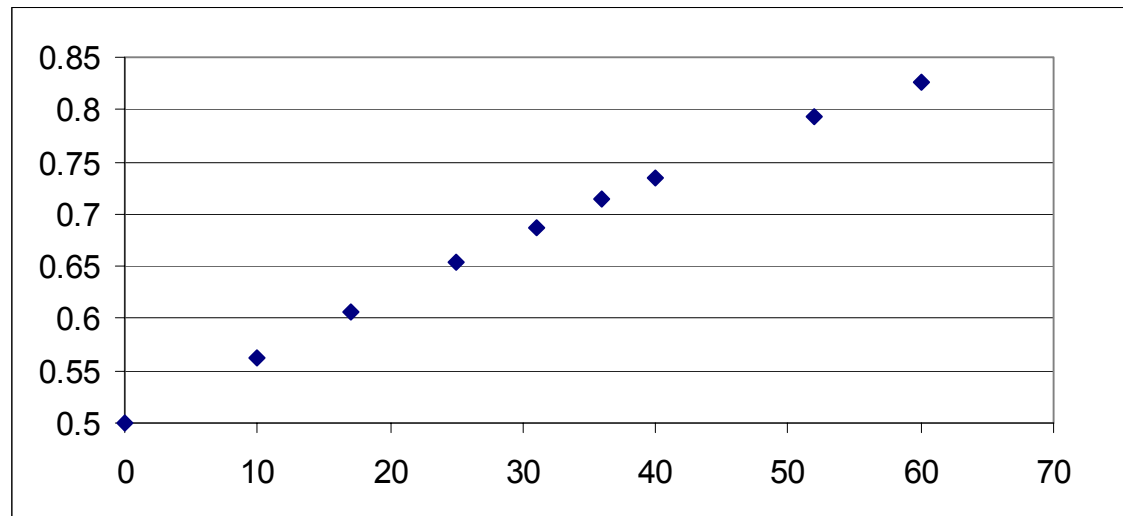
with it being cyclic in nature.

A plot of the solution to  $p' = (1/40)p(1-p)$ .



Maximum value  $p(60) = .817$

A plot of the solution to  $p^\Delta = (1/40)p(1-p)$ .  
on  $\{0, 10, 17, 25, 31, 36, 40, 52, \dots\}$



Maximum value  $p(60) = .825$

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