Commun. Appl. Anal., vol. 3, 1999, 271-281.

# Homogenization of an equation describing linear thin plates excited by piezopatches

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**ABSTRACT:** In this paper we consider the problem of homogenization of equations describing linear thin plates excited by actuators made of piezoelectric ceramics (see e.g. [1]). It is assumed that the number of actuators goes to infinity whereas their dimension tends to zero. The procedure of homogenization is based on the theory of two-scale convergence studied in [2]. The specific of the problem considered is the time dependence and the appearance of the forth spatial derivatives in the equation. A result of [3] about two-scale convergence of the second derivatives of subsequences of sequences bounded in $L_2(0,T;H_0^2(S))$ enables us to handle this case. The paper is illustrated by computer simulations that demonstrate a good approximation of solutions of the original equation by solutions of the homogenized equation if the number of piezoelectric patches is sufficiently large.

**KEY WORDS:** Linear thin plates, Piezoelectric actuators, Homogenization, Two-scale convergence.

**AMS (MOS) subject classification.** 35B27, 73K10, 73R05.

## 1. NOTATION

| | |
|---|---|
| $R$ | is the set of real numbers. |
| $R^n$ | is the real $n$-dimensional Euclidean space. |
| $S \subset R^2$ | is the domain occupied by the plate. |
| $S_{P_i} \subset S$ | is the domain occupied by the $i$th piezopatch. |
| $S_P = \bigcup_{i=1}^m S_{P_i}$ | is the domain occupied by all piezopatches. |
| $S_B = S \backslash S_P$ | is the domain occupied by the base material. |
| $Y = [0,1] \times [0,1]$ | is the unit square. |
| $\langle g \rangle = \int\int_Y g(y)dy$ | is the mean value of a function. |
| $H^2_\#(Y) \subset H^2(Y)$ | is the subspace of all periodic functions. |
| $H^2_\#(Y)/R$ | is the quotient space. |
| $C^\infty_\#(Y) \subset C^\infty(Y)$ | is the subspace of all periodic functions. |
| $Q = (0,T) \times S$ | |
| $\overset{\circ}{C}{}^\infty_T(Q) \subset C^\infty(Q)$ | is the subspace of all functions which vanish on $\partial S$ and at $t = T$ along with all derivatives. |

[1]This work is supported by DFG, Germany.

$\overset{\circ}{C}{}^{\infty}_{0,T}\,(Q; C^{\infty}_{\#}(Y))$ is the space of infinitely differentiable functions from $Q$ into $C^{\infty}_{\#}(Y)$ which vanish on $\partial S$, at $t = 0$, and $t = T$ along with all derivatives.

$H^2_T(0, T; L_2(S))$ is the subspace of all functions from $H^2(0, T; L_2(S))$ which vanish at $t = T$ along with the first derivatives.

## 2. PROBLEM SETTING

Consider a linear equation describing oscillations of a thin plate excited by patches made of a piezoelectric ceramic. For simplicity assume that the plate occupies a rectangular domain $S$ and the piezoelectric patches occupy rectangular domains $S_{P_i} \subset S$ (see Figure 1). It is assumed that the patches form a periodic structure of the period $\varepsilon$ so that the object is completely defined by $\varepsilon$. Denote $S_P := \cup_{i=1}^m S_{P_i}$ and $S_B = S \backslash S_P$.

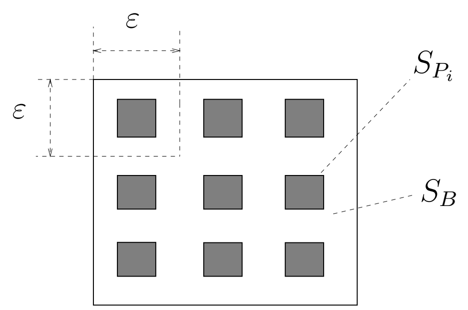


Figure 1. The plate $S$ excited by piezoelectric patches $S_{P_i}$.

The equation describing the model reads:

$$\rho \xi_{tt} + \Delta\Big(\gamma \Delta \xi\Big) = \sum_{i=1}^{m} v_i(t) \Delta I_{S_{P_i}}. \tag{1}$$

Here $\xi$ is the vertical displacement of the plate, $\Delta$ is the Laplace operator, $v_i(t)$ is the voltage applied to the $i$th piezoelectric patch $S_{P_i}$ , $I_{S_{P_i}}$ is the indicator function of the $i$th patch. The coefficients $\rho$ and $\gamma$ are piecewise constant functions defined as follows :

$$\rho = \begin{cases} \rho_P, & x \in S_{P_i}, \\ \rho_B, & x \in S_B, \end{cases} \qquad \gamma = \begin{cases} \gamma_P, & x \in S_{P_i}, \\ \gamma_B, & x \in S_B. \end{cases}$$

It is assumed that $\rho_P > 0, \quad \rho_B > 0, \quad \gamma_P > 0, \quad \gamma_B > 0$, and $v_i(\cdot) \in H^1(0, T)$.
Assume that the controls $v_i(\cdot)$ are being chosen as follows. A distribution $K(t, x) \in H^1(0, T; L_2(S))$ of the voltage over the whole plate $S$ is prescribed and we set

$$v_i(t) = \text{meas}(S_{P_i})^{-1} \iint_{S_{P_i}} K(t, x) dx.$$

The right-hand-side of (1) can be represented in the form: $\Delta\left(K_\varepsilon(t, x) I_{S_P}\right)$, where

$$K_\varepsilon(t, x) = \begin{cases} K(t, x), & x \in S_B, \\ \text{meas}(S_{P_i})^{-1} \iint_{S_{P_i}} K(t, x) dx, & x \in S_{P_i}. \end{cases}$$

It is obvious that $K_\varepsilon \to K$ in $H^1(0,T;L_2(S))$.

One can rewrite (1) as follows:

$$\rho\left(\tfrac{x}{\varepsilon}\right)\xi_{tt} + \Delta\Big(\gamma\left(\tfrac{x}{\varepsilon}\right)\Delta\xi\Big) = \Delta\Big(K_\varepsilon(t,x)I\left(\tfrac{x}{\varepsilon}\right)\Big), \tag{2}$$

where the coefficients $\rho, \gamma$, and $I$ are $1 \times 1$ periodic functions (the same notation for $\rho$ and $\gamma$ is held). Obviously, $0 < \min(\rho_P, \rho_B) \le \rho(y) \le \max(\rho_P, \rho_B)$, $0 < \min(\gamma_P, \gamma_B) \le \gamma(y) \le \max(\gamma_P, \gamma_B)$, and $0 \le I(y) \le 1$ for all $y \in R^2$.

Boundary and initial conditions are:

$$\xi|_{\partial S} = 0, \quad \partial\xi/\partial\vec{n}|_{\partial S} = 0, \quad \xi|_{t=0} = \xi_0, \quad \xi_t|_{t=0} = \xi_0'. \tag{3}$$

Note that (2) should be supplied with the following interface conditions:

$$[\xi] = 0, \quad \left[\frac{\partial\xi}{\partial\vec{n}}\right] = 0, \quad [\gamma\Delta\xi] = 0, \quad \left[\frac{\partial}{\partial\vec{n}}\gamma\Delta\xi\right] = 0 \tag{4}$$

that hold on the boundary between $S_P$ and $S_B$ because of the integration by parts when deriving (2) from a weak formulation. Here $[\cdot]$ denotes the jump of a function on the boundary between $S_P$ and $S_B$. We do not pay any attention to these conditions because we go back to the weak formulation.

We say that a function $\xi^\varepsilon \in L_2(0,T;H_0^2(S))$ is a solution of the problem (2)-(4) if the following equality:

$$\int_0^T \int\int_S \Big(\rho\left(\tfrac{x}{\varepsilon}\right)\xi^\varepsilon\varphi_{tt} + \gamma\left(\tfrac{x}{\varepsilon}\right)\Delta\xi^\varepsilon\Delta\varphi - K_\varepsilon(t,x)I\left(\tfrac{x}{\varepsilon}\right)\Delta\varphi\Big)dxdt + \tag{5}$$

$$+\int\int_S \rho\left(\tfrac{x}{\varepsilon}\right)(\xi_0'\varphi(0,x) - \xi_0\varphi_t(0,x))dx = 0$$

holds for all $\varphi \in H_T^2(0,T;L_2(S)) \cap L_2(0,T;H_0^2(S))$.

**PROPOSITION 1.** If $\xi_0 \in H_0^2(S)$, $\xi_0' \in L_2(S)$, and $K \in H^1(0,T;L_2(S))$, then, for any $\varepsilon > 0$, the equation (5) has a unique solution $\xi^\varepsilon$ such that
$(\xi^\varepsilon, \xi_t^\varepsilon) \in C([0,T];H_0^2(S)) \times C([0,T];L_2(S))$ and

$$\|\xi^\varepsilon\|^2_{C([0,T];H_0^2(S))} + \|\xi_t^\varepsilon\|^2_{C([0,T];L_2(S))} \le$$

$$C\left(\|\xi_0\|^2_{H_0^2(S)} + \|\xi_0'\|^2_{L_2(S)} + \|K\|^2_{C([0,T];L_2(S))} + \|K_t\|^2_{L_2(0,T;L_2(S))} + \omega(\varepsilon)\right),$$

with $C$ independent from $\varepsilon$ and $\omega(\varepsilon) \to 0$ as $\varepsilon \to 0$.

The proof of this proposition can be found in [4]. ■

Thus, $\|\xi^{\varepsilon}\|_{L_\infty(0,T;H_0^2(S))} \leq C$ with $C$ independent from $\varepsilon$. Therefore, any sequence $\xi^{\varepsilon_j}$ contains a weak-$*$ converging subsequence in $L_\infty(0,T;H_0^2(S))$. Now we derive an equation that defines limit functions of such subsequences (the effective equation). We will show that this equation has a unique solution and that the coefficients of the equation are independent from the choice of the subsequence $\xi^{\varepsilon_j}$. This yields that the sequence $\xi^{\varepsilon}$ of solutions of (5) converges weak-$*$ in $L_\infty(0,T;H_0^2(S))$ to the solution of the effective equation. The similar arguments show that $\xi_t^{\varepsilon}$ converges weak-$*$ in $L_\infty(0,T;L_2(S))$ to the time derivative of the solution of the effective equation. Then, using Corollary 4 of Simon [5] and Theorem 16.1 of Lions [6], we conclude that $\xi^{\varepsilon}$ converges strongly in $C([0,T];H_0^{2-\alpha}(S))$ for any positive real $\alpha$. In particular, $\xi^{\varepsilon}$ converges uniformly on $[0,T]\times\bar{S}$.

Let us reproduce the definition of two-scale convergence of functions depending on additional parameters (Definition 6.8 of Haller [3]): Let $u_\varepsilon \in L_2(Q)$, $\quad u_0 \in L_2(Q\times Y)$. It is said that $u_\varepsilon \overset{2-scale}{\longrightarrow} u_0$, if

$$\lim_{\varepsilon\to 0}\int_0^T\int\int_S u_\varepsilon(t,x)\psi(t,x,x/\varepsilon)dxdt = \int_0^T\int\int_S\int\int_Y u_0(t,x,y)\psi(t,x,y)dydxdt$$

for all $\psi \in \overset{\circ}{C}{}^{\infty}_{0,T}(Q;C^{\infty}_{\#}(Y))$.

It is proved (Theorem 6.15 of Haller [3]) that all properties of two-scale convergence hold, if the test functions in the above definition are replaced by more general test functions of the form: $\psi(t,x,y) = \alpha(t,x)\beta(y)\sigma(t,x,y)$, where $\alpha \in L_\infty(Q)$, $\beta(y) \in L_{\infty\#}(Y)$, and $\sigma \in C^\infty(Q;C^\infty_{\#}(Y))$ (not necessary vanishes).

To obtain the effective equation, set

$$\varphi(t,x) = \eta(t,x) + \varepsilon^2\psi(t,x,\tfrac{x}{\varepsilon}),$$

where $\eta(t,x) \in \overset{\circ}{C}{}^{\infty}_{T}(Q)$ and $\psi(t,x,y) \in \overset{\circ}{C}{}^{\infty}_{0,T}(Q;C^{\infty}_{\#}(Y))$. Substituting this function into (5) yields

$$\begin{gathered}
\int_0^T\int\int_S\Big(\rho\left(\tfrac{x}{\varepsilon}\right)\xi^{\varepsilon}[\eta_{tt}(t,x)+\varepsilon^2\psi_{tt}(t,x,\tfrac{x}{\varepsilon})]+ \\
\gamma\left(\tfrac{x}{\varepsilon}\right)\Delta\xi^{\varepsilon}[\Delta\eta(t,x)+\Delta_y\psi(t,x,\tfrac{x}{\varepsilon})+\varepsilon^2(\ldots)]- \\
K(t,x)I\left(\tfrac{x}{\varepsilon}\right)[\Delta\eta(t,x)+\Delta_y\psi(t,x,\tfrac{x}{\varepsilon})+\varepsilon^2(\ldots)]+ \\
[K(t,x)-K_\varepsilon(t,x)]I\left(\tfrac{x}{\varepsilon}\right)[\Delta\eta(t,x)+\Delta_y\psi(t,x,\tfrac{x}{\varepsilon})+\varepsilon^2(\ldots)]\Big)dxdt+ \\
\int\int_S\rho\left(\tfrac{x}{\varepsilon}\right)(\xi_0'\eta(0,x)-\xi_0\eta_t(0,x))dx = 0.
\end{gathered} \tag{6}$$

The symbol $(\ldots)$ denotes terms with the multipliers 1 and $1/\varepsilon$. Due to Theorem 6.12 of Haller [3], there exist: $\varepsilon_j$, $\xi(t,x) \in L_2(0,T;H_0^2(S))$, and $u(t,x,y) \in L_2(Q;H_\#^2(Y))$ such that

$$\xi^{\varepsilon_j} \overset{two-scale}{\longrightarrow} \xi,$$

$$\Delta\xi^{\varepsilon_j} \overset{two-scale}{\longrightarrow} \Delta\xi + \Delta_y u.$$

Considering $\rho(y)$, $\gamma(y)$, $\gamma(y)\Delta_y\psi(t,x,y)$, $I(y)$, and $I(y)\Delta_y\psi(t,x,y)$ as test functions, one can pass to the two–scale limit in (6). Under the observation that $\varepsilon^2(\ldots) \overset{two-scale}{\longrightarrow} 0$ and $[K(t,x)-K_\varepsilon(t,x)]I\left(\frac{x}{\varepsilon}\right)[\Delta\eta(t,x)+\Delta_y\psi(t,x,\frac{x}{\varepsilon})+\varepsilon^2(\ldots)] \overset{two-scale}{\longrightarrow} 0$, one obtains:

$$\int_0^T \iint_S \iint_Y \Big(\rho(y)\xi\eta_{tt}(t,x)+$$
$$\gamma(y)[\Delta\xi+\Delta_y u][\Delta\eta(t,x)+\Delta_y\psi(t,x,y)]-$$
$$K(t,x)I(y)[\Delta\eta(t,x)+\Delta_y\psi(t,x,y)]\Big)dydxdt+$$
$$\iint_S \iint_Y \rho(y)(\xi_0'\eta(0,x)-\xi_0\eta_t(0,x))dydx = 0.$$

Because of the arbitrary choice of the functions $\eta$ and $\psi$ the following two equations must hold:

$$\int_0^T \iint_S \iint_Y \Big(\rho(y)\xi\eta_{tt}+\gamma(y)[\Delta\xi+\Delta_y u]\Delta\eta - K(t,x)I(y)\Delta\eta\Big)dydxdt+ \tag{7}$$
$$\iint_S \iint_Y \rho(y)(\xi_0'\eta(0,x)-\xi_0\eta_t(0,x))dydx = 0,$$

$$\int_0^T \iint_S \iint_Y \Big(\gamma(y)[\Delta\xi+\Delta_y u]\Delta_y\psi - K(t,x)I(y)\Delta_y\psi\Big)dydxdt = 0. \tag{8}$$

The second equation enables us to find the auxiliary function $u$. The first one becomes the effective equation after substituting $u$.

Because of the superposition principle and the symmetry of $\gamma$, one can seek $u$ in the form (one auxiliary function $N$ is sufficient):

$$u(t,x,y) = N(y)\Delta\xi + M(y)K(t,x), \tag{9}$$

where $N(y)$ and $M(y)$ are unknown functions. Substitution of (9) into (8) gives

$$\int_0^T \iint_S \iint_Y \Big[\gamma(y)(1+\Delta_y N)\Delta\xi + K(t,x)(\gamma(y)\Delta_y M - I(y))\Big]\Delta_y\psi dydxdt = 0.$$

Because of the superposition principle, one can find $M$ and $N$ separately solving the following two equations:

(10) $$\int_0^T \iint_S \iint_Y \gamma(y)(1+\Delta_y N)\Delta\xi\Delta_y\psi dydxdt = 0,$$

(11) $$\int_0^T \iint_S \iint_Y K(t,x)(\gamma(y)\Delta_y M - I(y))\Delta_y\psi dydxdt = 0.$$

Consider (10). Taking $\psi$ of the form $\psi(t,x,y) = \psi_1(t,x)\psi_2(y)$, one obtains

(12) $$\int_0^T \iint_S \Delta\xi\psi_1 dxdt \cdot \iint_Y \gamma(y)(1+\Delta_y N)\Delta_y\psi_2 dy = 0.$$

If the first multiplier in (12) is equal to zero for any admissible $\psi_1$, then $\Delta\xi \equiv 0$ and (10) is satisfied by any $N$. Hence one can ignore this multiplier to find $N$ from the equation

(13) $$\iint_Y \gamma(y)(1+\Delta_y N)\Delta_y\psi_2 dy = 0, \quad \forall\psi_2 \in H^2_\#(Y).$$

Consider (11). Taking $\psi$ of the form $\psi(t,x,y) = \psi_1(t,x)\psi_2(y)$, one obtains

(14) $$\int_0^T \iint_S K(t,x)\psi_1(t,x)dxdt \cdot \iint_Y (\gamma(y)\Delta_y M - I(y))\Delta_y\psi_2 dy = 0.$$

If the first multiplier in (14) is equal to zero for any admissible $\psi_1$, then $K \equiv 0$ and (11) is satisfied by any $M$. Hence one can ignore this multiplier to find $M$ from the equation

(15) $$\iint_Y (\gamma(y)\Delta_y M - I(y))\Delta_y\psi dy = 0, \quad \forall\psi_2 \in H^2_\#(Y).$$

The following simple facts will be used.

**PROPOSITION 2.** For any $f \in H^2_\#(Y)$, holds:

$$\iint_Y \Delta_y f(y)dy = 0.$$

PROOF. Applying the Gauss formula, we get:

$$\iint_Y \Delta_y f(y)dy = -\int_0^1 f_{y_2}(y_1,0)dy_1 + \int_0^1 f_{y_2}(y_1,1)dy_1 - \int_0^1 f_{y_1}(0,y_2)dy_2 + \int_0^1 f_{y_1}(1,y_2)dy_2 = 0$$

because of periodicity of the first derivatives of $f$.■

**PROPOSITION 3.** Let $f \in L_2(Y)$ and $\iint_Y f dy = 0$. Then the equation

$$\Delta_y v = f, \quad \text{for a.e. } y \in Y$$

has a unique solution $v \in H^2_\#(Y)/R$.

PROOF. Consider the equation:

$$-\iint\limits_Y \nabla v \nabla \eta dy = \iint\limits_Y f \eta dy, \quad \forall \eta \in H^1_\#(Y), \tag{16}$$

where $f \in L_2(Y)$ and $\iint\limits_Y f dy = 0$. The left-hand-side of (16) is a scalar product of $H^1_\#(Y)/R$ and the right-hand-side is a continuous w.r.t. this scalar product linear functional on $H^1_\#(Y)/R$. Therefore, there is a function $v \in H^1_\#(Y)$ satisfying (16). With arguments like to those of Mikhailov [7], one can prove that $v \in H^2(Y)$: Consider $\eta = \delta^i_{-h}\tilde{\eta}$, $i = 1, 2$, where $\delta^i_{-h}$ is the finite-difference fraction in $y_i$, $i = 1, 2$, with the step $-h$, and $\tilde{\eta} \in H^1_\#(Y)$ is an arbitrary function. One obtains from (16):

$$\iint\limits_Y \nabla \delta^i_h v \nabla \tilde{\eta} dy = -\iint\limits_Y f \delta^i_{-h} \tilde{\eta} dy, \quad \forall \tilde{\eta} \in H^1_\#(Y), \; i = 1, 2.$$

Using that $\|\delta^i_{-h}\tilde{\eta}\|_{L_2(Y)} \leq G\|\nabla\tilde{\eta}\|_{L_2(Y)}$, where $G$ does not depend from $h$, and taking $\tilde{\eta} = \delta^i_h v$, we obtain:

$$\|\nabla \delta^i_h v\|_{L_2(Y)} \leq G\|f\|_{L_2(Y)}, \; i = 1, 2.$$

Therefore, $v \in H^2(Y)$ by Theorem 3 of § 3 of Chapter 3 of Mikhailov [7]. The application of the Gauss formula to (16) gives:

$$\iint\limits_Y (\Delta v - f)\eta dy + \int_0^1 v_{y_2}(y_1, 0)\eta(y_1, 0)dy_1 - \int_0^1 v_{y_2}(y_1, 1)\eta(y_1, 1)dy_1$$

$$+ \int_0^1 v_{y_1}(0, y_2)\eta(0, y_2)dy_2 - \int_0^1 v_{y_1}(1, y_2)\eta(1, y_2)dy_2 = 0, \quad \forall \eta \in H^1_\#(Y).$$

Since $H^1_\#(Y)$ is dense in $L_2(Y)$, one concludes that

$$\Delta v = f, \quad \text{for a.e. } y \in Y$$

and

$$v_{y_1}(0, y_2) = v_{y_1}(1, y_2), \quad v_{y_2}(y_1, 0) = v_{y_2}(y_1, 1), \quad \text{for a.e. } y_1, y_2 \in [0, 1]$$

because of the periodicity of $\eta$. Therefore, $v \in H^2_\#(Y)$.■

**PROPOSITION 4.** The equation (13) is equivalent to the following one:

$$\gamma(y)(1 + \Delta_y N) = \langle 1/\gamma \rangle^{-1}, \quad \text{for a.e. } y \in Y. \tag{17}$$

The equation (17) has a unique solution $N \in H^2_\#(Y)/R$.

PROOF. The last claim of the proposition follows from Proposition 3. Moreover, if (17) is true, then (13) holds due to Proposition 2. Now assume (13) holds. Hence, for any constant $C$, we have due to Proposition 2:

$$\iint\limits_Y [\gamma(y)(1 + \Delta_y N) - C]\Delta_y \psi_2 dy = 0, \quad \forall \psi_2 \in H^2_\#(Y). \tag{18}$$

Let $C = \langle \gamma(y)(1+\Delta_y N)\rangle$. Then the equation

$$\Delta_y \psi_2 = \gamma(y)(1+\Delta_y N) - C, \quad \text{for a.e. } y \in Y, \tag{19}$$

has a solution $\psi_2 \in H^2_\#(Y)$ due to Proposition 3. Substituting such a $\psi_2$ into (18), we conclude that

$$\gamma(y)(1+\Delta_y N) = C, \quad \text{for a.e. } y \in Y.$$

Dividing this equation over $\gamma(y)$ and integrating over $Y$, we obtain using Proposition 2 that $C = \langle 1/\gamma\rangle^{-1}$. ■

**PROPOSITION 5.** The equation (15) is equivalent to the following one:

$$\gamma(y)\Delta_y M - I(y) = -\langle 1/\gamma\rangle^{-1}\langle I/\gamma\rangle, \quad \text{for a.e. } y \in Y. \tag{20}$$

The equation (20) has a unique solution $M \in H^2_\#(Y)/R$.

The proof of this proposition is identical to the one of Proposition 4.■

Now we can state the effective equation. To this end, we substitute (9) into (7) to obtain

$$\int_0^T \int\int_S \Big( \langle\rho\rangle \xi \eta_{tt} + \langle \gamma(1+\Delta_y N)\rangle \Delta\xi\Delta\eta + \langle \gamma\Delta_y M - I\rangle K(t,x)\Delta\eta \Big) dxdt + \tag{21}$$

$$\int\int_S \langle\rho\rangle(\xi_0' \eta(0,x) - \xi_0 \eta_t(0,x))dx = 0$$

Since $N$ and $M$ are solutions of (17) and (20), respectively, we conclude that

$$\langle \gamma(1+\Delta_y N)\rangle = \langle 1/\gamma\rangle^{-1},$$
$$\langle \gamma\Delta_y M - I\rangle = -\langle 1/\gamma\rangle^{-1}\langle I/\gamma\rangle.$$

Finally, we have

$$\int_0^T \int\int_S \Big( \langle\rho\rangle \xi \eta_{tt} + \langle 1/\gamma\rangle^{-1} \Delta\xi\Delta\eta - \langle 1/\gamma\rangle^{-1}\langle I/\gamma\rangle K(t,x)\Delta\eta \Big) dxdt + \tag{22}$$

$$+ \int\int_S \langle\rho\rangle(\xi_0' \eta(0,x) - \xi_0 \eta_t(0,x))dx = 0.$$

The classical form of the effective equation reads

$$\langle\rho\rangle \xi_{tt} + \langle 1/\gamma\rangle^{-1}\Delta^2\xi = \langle 1/\gamma\rangle^{-1}\langle I/\gamma\rangle \Delta K(t,x) \tag{23}$$

with boundary and initial conditions (3) and without any interface conditions.

## 4. SIMULATION

The simulation was done with the following parameters: $S$ is the unit square, $S_{P_i}$ is the $\varepsilon/2$-edge square centered w.r.t. the corresponding structural cell (see Figure 1). The other values are:

$$\rho_P = 1.5, \quad \rho_B = 1, \quad \gamma_P = 0.05, \quad \gamma_B = 0.03, \quad \varepsilon = 1/6,$$
$$K(t, x_1, x_2) = \sin 5t \cos 5(x_1 + x_2).$$

The initial values were equal to zero. The Bogner-Fox-Schmit finite elements (see Ciarlet [8]) were applied. The number of finite elements was equal to 24×24. Each structural cell (see Figure 1) occupies 4×4 elements Each piezopatch occupies 2×2 elements. Each of the figures below shows the solution of (2), the solution of the effective equation (23), and their difference at the time indicated.

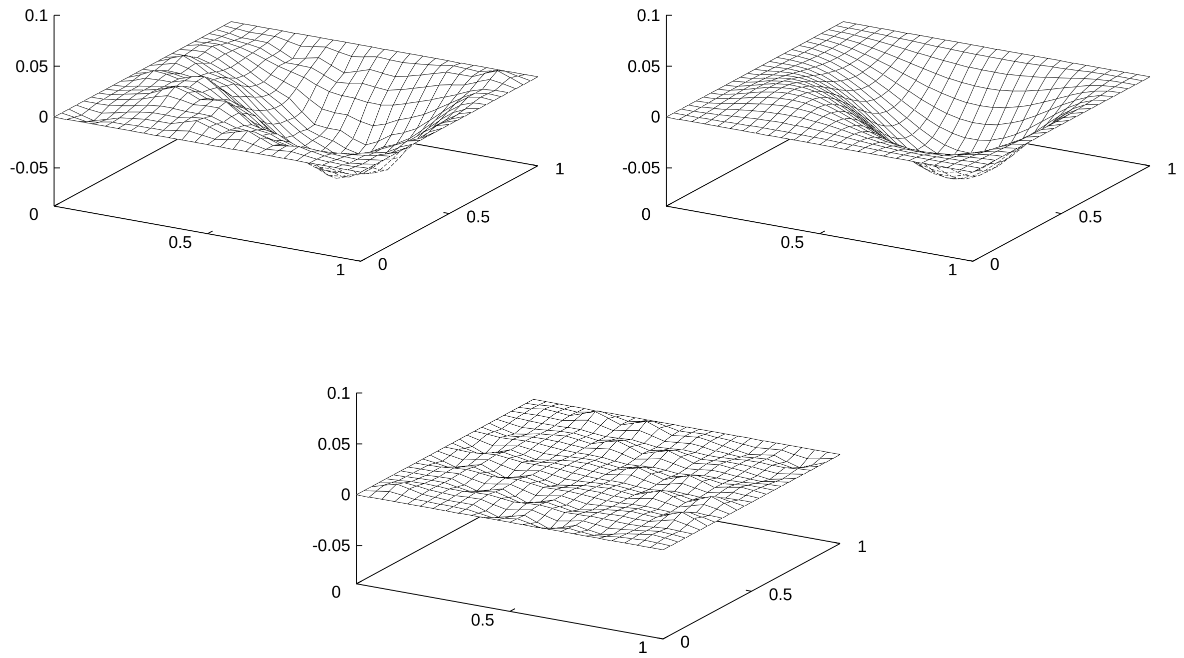


Figure 2. $t = 0.5$

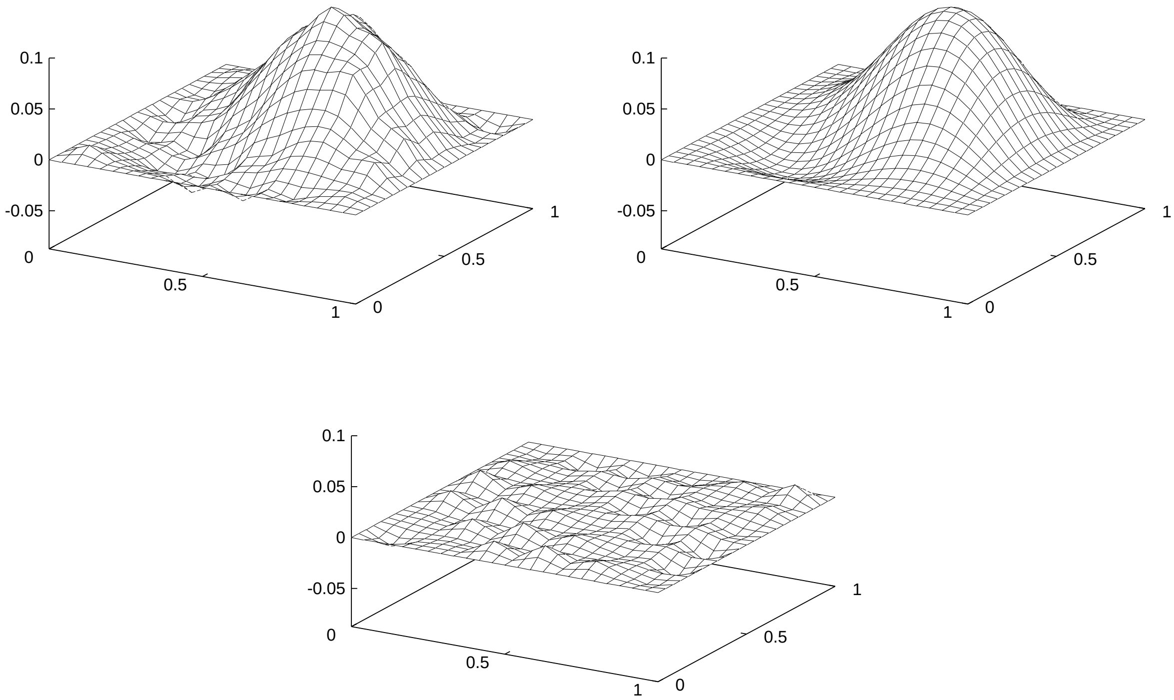


Figure 3. $t = 1$

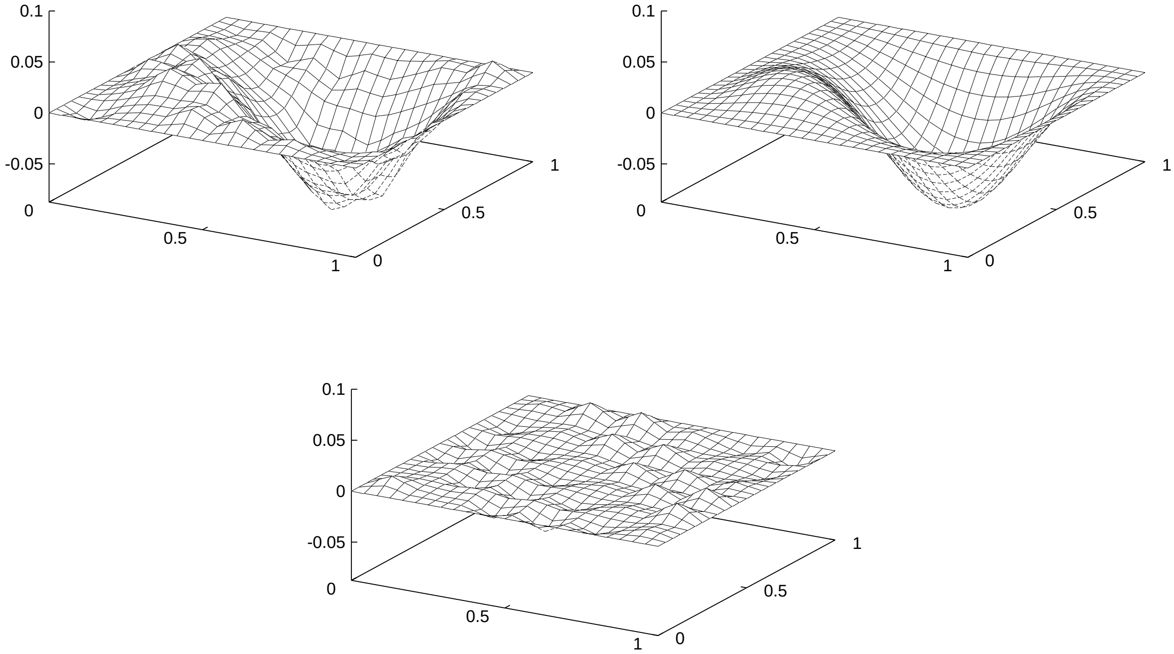


Figure 4. $t = 1.5$

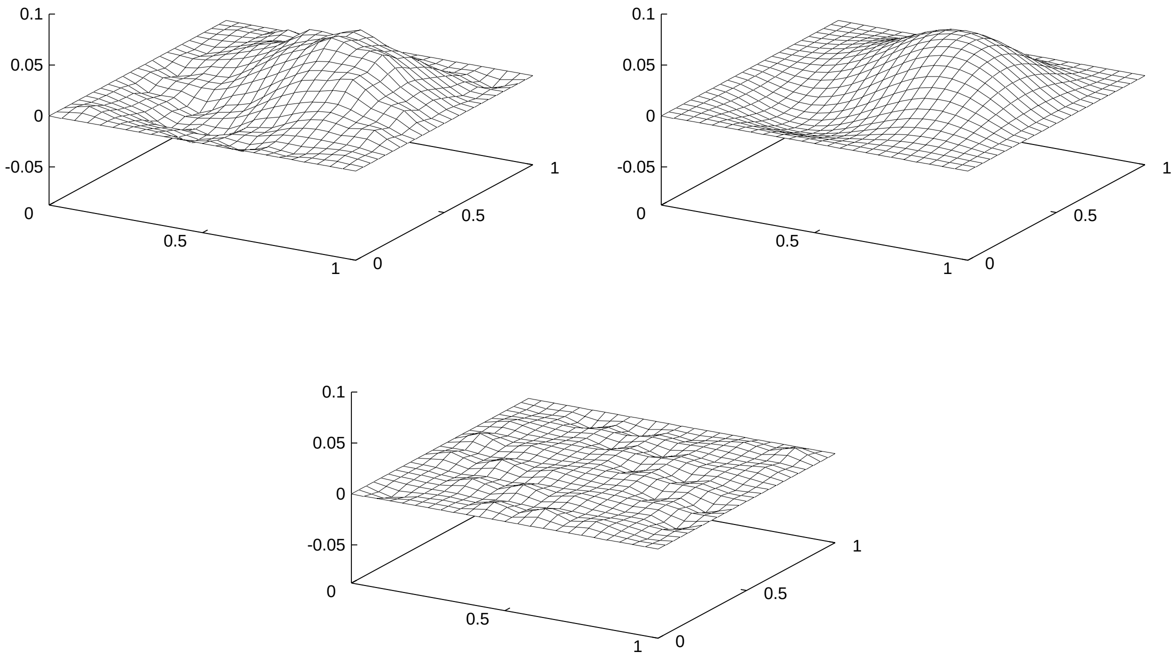


Figure 5. $t = 2$

## ACKNOWLEDGMENTS

The author thanks Prof. Dr. K.-H. Hoffmann for the statement of the problem. The author is also grateful Dr. H. Haller for his important advices throughout the work.

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