Reachable Sets for Some Simple Models of the Car’s Motion

Andrey Fedotov

Institute of Mathematics and Mechanics, S. Kovalevskaya str. 16,
Ekaterinburg, 620219, Russia
e-mail: fedotov@imm.uran.ru

Valery Patsko

Institute of Mathematics and Mechanics, S. Kovalevskaya str. 16,
Ekaterinburg, 620219, Russia
e-mail: patsko@imm.uran.ru

Varvara Turova

Mathematical Centre, Munich Technical University, Boltzmannstr. 3,
Garching bei München, Germany
e-mail: turova@ma.tum.de

1. The simplest car dynamics written in dimensionless variables are

\[ \dot{x} = \sin \theta, \quad \dot{y} = \cos \theta, \quad \dot{\theta} = u; \quad |u| \leq 1. \] (1)

The motion trajectories in the plane \(x, y\) are curves of bounded curvature. The paper [5] published by A. A. Markov in 1889 considers four optimization problems related to curves of bounded curvature. The first problem [5], p. 250, can be interpreted as a time-optimal control problem for car dynamics (1). Also, the main theorem [2], p. 515, of the paper by L. E. Dubins (1957) allows an interpretation in the context of time-optimal problem for such a car. In works on theoretical robotics [4], an object with dynamics (1) is called Dubins’ car. Model (1) is often utilized in differential game problem formulations [3].

Next in complexity is the car model by Reeds and Shepp [4, 6]:

\[ \dot{x} = w \sin \theta, \quad \dot{y} = w \cos \theta, \quad \dot{\theta} = u; \quad |u| \leq 1, |w| \leq 1. \] (2)

Control \(u\) changes the angular velocity, control \(w\) is responsible for instantaneous changes of the linear velocity magnitude. In particular, the car can instantaneously change its motion direction to the opposite one. The angle \(\theta\) is the angle between the direction of the linear velocity magnitude and the direction of the forward motion of the car.

It is natural to consider control dynamics where the control \(w\) is from the interval \([a, 1]\):

\[ \dot{x} = w \sin \theta, \quad \dot{y} = w \cos \theta, \quad \dot{\theta} = u; \quad |u| \leq 1, w \in [a, 1]. \] (3)
Here $a \in [-1, 1]$ is the parameter of the problem. If $a = 1$, Dubins’ car is obtained; if $a = -1$, Reeds and Shepp’s car appears.

2. Let $x_0$, $y_0$ be the initial geometric position and $\theta_0$ be the initial angle at time $t_0 = 0$. Reachable set $G(t)$ (or $\mathcal{G}(t)$) at given time $t$ (by time $t$) in the plane $x, y$ is the set of all geometric positions which can be reached at time $t$ (on the time interval $[0, t]$) from the starting point $x_0$, $y_0$, $\theta_0$ using admissible measurable controls.

An analytically based description of reachable sets $G(t)$ for system (1) is given in [1]. The set $G(t)$ for system (2) is investigated in [7]. The structure of reachable sets $\mathcal{G}(t)$ for system (1) is also well-known. For system (2), equality $\mathcal{G}(t) = G(t)$ holds.

In this paper, reachable sets $G(t)$ and $\mathcal{G}(t)$ for system (3) are investigated numerically using an algorithm developed by the authors. The dependence of the form of these sets on the parameter $a$ is studied.

3. The construction of reachable sets in three-dimensional space of variables $x$, $y$, $\theta$ is an essentially more complicated problem. In the talk, numerically computed three-dimensional reachable sets at given time for system (1) will be presented.

References


