Motivation

Development of an acoustic wave sensor for biological and medical applications (microbalance)

Operation principles

1. Excitation of acoustic shear waves due to piezoelectric properties of a substrate
2. Selective binding of the biomolecules from a contacting liquid due to aptamers
3. Mass estimation through the measurement of the phase shift in the electric signal

Methods of modelling

Direct modelling with finite elements (resources consuming because of short wave lengths)

Dispersion relations (dependence of the wave propagation velocity on the frequency) for multi-layered anisotropic structures

Description of wave fronts using Hamilton-Jacobi equations

Pontryagin’s maximum principle

\[
\dot{v} = u, \quad u \in \mathbb{R}^2, \quad T = \inf \left\{ \int_0^T \frac{|u|}{V(x, u)} dt : u \in C_{ad}^{1,2} \right\}
\]

\[p = \text{adjoint vector}, \quad \max_{u \in \mathbb{R}} |u| = \max_{v \in \mathbb{R}} v = 1 \]

\[V = \text{wave surface} \]

Applicable if \( F_p \) is a well-defined function

Waves description using optimality principles

Characteristic surfaces

\[ \mathbf{V}_n = n \quad \text{wave vector} \]

\[ \mathbf{F}_\nu = \nu |F| \quad \text{phase velocity} \]

The energy velocity is normal to the slowness surface at all points

Complicated structure of wave surfaces

Example of a wave surface for a crystal with cubic symmetry

Variation of the curvature of the slowness surface can yield multiple values for the energy velocity (swallow tails in the wave surface)

Usage of differential games

\[ \dot{x} = f(x, u, v), \quad u \in P, \quad v \in Q \]

The first player minimizes and the second player maximizes the time of attaining a given terminal set \( M \)

Value function \( J(x) \) satisfies the following Hamilton-Jacobi equation

\[ \min_{u \in P} \max_{v \in Q} \left\{ \frac{-\partial J(x)}{\partial t} + f(x, u, v) \right\} = 0 \]

at all points \( x \) where \( J(x) \) is differentiable. In other points, this equation holds in a viscosity sense. If we find a differential game with the dynamics that provides the equality

\[ -\min_{u \in P} \max_{v \in Q} \left\{ \frac{-\partial J(x)}{\partial t} + f(x, u, v) \right\} = -1 \]

then level sets of \( t \) (i.e. \( J(x, t) = c \)) yield wave fronts in the wave propagation problem.

Application to the propagation of surface acoustic waves

Multi-layered structure

Velocity contour for shear surface acoustic waves

Application of the phase velocity contour

Choice of \( P \)

Choice of \( Q \)

The result

Numerical solution to the differential game

\[ \dot{v} = u, \quad u \in P, \quad v \in Q \]

\( u \) minimizes and \( v \) maximizes the time of attaining the terminal set \( M \)

Backward step-by-step computation of the attainability set on the interval \( [0,T] \) with a step \( \Delta t \)

Investigation of singular lines

\[ \mathbf{R}(x, \lambda, \mu) \]

\[ \mathbf{B}(x, \lambda, \mu) \]

\[ S = \max \{S^+, S^-\} \]

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