Dispersion relations for acoustic waves in heterogeneous multi-layered structures contacting with fluids

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Abstract

The paper describes a method and a computer application for the computation of the velocity of acoustic waves excited in complicated multi-layered structures consisting of anisotropic piezoelectric and isotropic layers. The structure assumes to be unbounded in the lateral directions. The top and bottom layers are either semi-infinite in the vertical direction or they contact with media such as fluids, gases or vacuum. A special homogenization technique enables to account for bristle-like layers contacting with a fluid. The program is supplied with a user friendly graphical interface and can be useful for researchers working on acoustic sensors.

Key words: Multi-layered structures, Surface acoustic waves, Dispersion relations, Homogenization, Biosensor

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1 Introduction

In contrast to acoustic waves in bulk materials, the wave velocity in multi-layered structures depends on the wave frequency because of the interaction between the layers with different acoustic properties. Therefore, one can speak about dispersion relations that express the connection between the propagation velocity and the wave frequency.

Computation of dispersion relations for multi-layered anisotropic structures is very important for many applications, which makes it the topic of many papers (see e.g. [1–3]).

Though the proposed in this paper algorithm and the program can be applied for a wide range of problems, the primarily pursued application is the modeling of a biosensor that serves for the detection and quantitative measurement of microscopic amounts of biological substances solved in a liquid (see e.g. [4–7]). The biosensor is a multi-layered structure consisting basically of anisotropic piezoelectric and isotropic layers. The surface of the top layer is covered with tiny receptors (aptamers) which can bind specific proteins from the contacting fluid. The operating principle is based on the excitation and detection of horizontally polarized acoustic shear Love waves: as the proteins adhere to the aptamers a new layer is being formed, which changes the velocity of shear waves propagating along the sensor surface.

The algorithm is based on the construction of travelling wave solutions to equations describing deformations in the layers. The wave velocity is computed from the fitting of appropriate mechanical and (if applicable) electrical conditions on the interfaces between the layers. Mechanical conditions express the equilibrium of normal pressures and, depending on the nature of the contacting layers, either the continuity of the displacement field or the continuity of the velocity field. Electrical relations express the jump conditions for the
electric field and electric displacement.

A measure of the inconsistence in the interface conditions is expressed then by means of a non-negative real function (fitting function) whose roots are feasible values of the wave velocity.

A new feature of the method developed is accounting for the extremely thin periodic structure consisting of protein molecules adhered to the atamers and contacting with the liquid. This is being done using the mathematical theory of homogenization. Note that such a feature enables to explain a very high sensitivity of the biosensor observed in experiments.

2 Mathematical model

2.1 Description of acoustic waves in solid multi-layered structures

A solid multi-layered structure consists of a finite number of stacked layers as it is depicted in Figure 1. The top and bottom layers are either semi-infinite or their outer surfaces are in contact with some media such as liquids, gases or vacuum. All the layers are infinite in $x_1$ and $x_2$ directions. Besides mechanical properties, each layer can also possess piezoelectric, dielectric or conductivity properties. In this section, we consider only piezoelectric anisotropic layers as the most general case. The relations for dielectric and conductor layers look simpler.

![Fig. 1. A sample structure.](image_url)
The constitutive relations for piezoelectric layers in the case of small deformations are of the form:

\[ \sigma_{ij} = C_{ijkl} \varepsilon_{kl} - e_{kij} E_k, \quad D_i = \epsilon_{ij} E_j + e_{ikl} \varepsilon_{kl}. \]

Here, \( \sigma_{ij} \) and \( \varepsilon_{kl} \) are the stress and the strain tensors, \( D_i \) and \( E_i \) denote the electric displacement and the electric field; \( e_{kij}, e_{kij}, \) and \( C_{ijkl} \) denote the material dielectric tensor, the stress piezoelectric tensor, and the elastic stiffness tensor, respectively. The corresponding governing equations for the displacements and the electric potential are:

\[ \rho u_{i tt} - C_{ijkl} \frac{\partial^2 u_i}{\partial x_j \partial x_k} + e_{kij} \frac{\partial^2 \phi}{\partial x_k \partial x_j} = 0, \quad (1) \]

\[ \epsilon_{ij} \frac{\partial^2 \phi}{\partial x_i \partial x_j} + e_{ikl} \frac{\partial^2 u_l}{\partial x_k \partial x_j} = 0, \quad (2) \]

where \( u_i, i = 1, 2, 3, \) are components of the displacement vector, \( \phi \) is the electric potential such that \( E_i = \partial \phi / \partial x_i, \) and \( \rho \) is the density of the piezoelectric.

Note that the stress piezoelectric tensor \( e_{kij} \) vanishes for purely dielectric layers and, therefore, elasticity equation (1) and equation (2) describing the electric potential are decoupled:

\[ \rho u_{i tt} - C_{ijkl} \frac{\partial^2 u_i}{\partial x_j \partial x_k} = 0, \]

\[ \epsilon_{ij} \frac{\partial^2 \phi}{\partial x_i \partial x_j} = 0. \quad (3) \]

In conductor layers like metals there is no electric field and, therefore, equation (3) for the electric potential disappears.

We are looking for a solution describing a plain wave propagating in \( x_1 \) direction, which means that the displacement field and the electric potential are of the form:

\[ u_i(x_1, x_3) = a_i(x_3) \cos(\kappa x_1 - \omega t) + b_i(x_3) \sin(\kappa x_1 - \omega t), \quad (4) \]

\[ \phi(x_1, x_3) = f(x_3) \cos(\kappa x_1 - \omega t) + g(x_3) \sin(\kappa x_1 - \omega t), \quad (5) \]
where $\kappa$ is the wave number and $\omega$ is the circular frequency. Substituting (4) and (5) into system (1), (2) and equating the coefficients on cos and sin, we obtain a system of ordinary linear differential equations. Using the state vector

$$\vec{q} := (a_1, a_2, a_3, b_1, b_2, b_3, f, g, \dot{a}_1, \dot{a}_2, \dot{a}_3, \dot{b}_1, \dot{b}_2, \dot{b}_3, \dot{f}, \dot{g}),$$

(6) this system can be rewritten in the normal form as follows:

$$\dot{\vec{q}} = A \vec{q},$$

(7)

where $A$ is the corresponding matrix, the dot denotes the differentiation with respect to the variable $\tilde{x}_3 = \kappa x_3$. Let $\lambda_1, ..., \lambda_{16}$ and $\vec{h}_1, ..., \vec{h}_{16}$ be eigenvalues and eigenvectors of $A$. Due to the nature of the phenomena, polynomial solutions are not expected, which means that all the eigenvectors corresponding to a $k$-multiple eigenvalue form a $k$-dimensional eigensubspace. Therefore, the general solution of (7) is of the form:

$$\vec{q}(x_3) = \sum_{j=1}^{16} D_j \vec{h}_j e^{\lambda_{ij} \kappa x_3},$$

(8)

where $D_j$ are arbitrary coefficients. If the layer is semi-infinite in the $x_3$ (resp. $-x_3$) direction, only the terms decreasing towards $x_3$ (resp. $-x_3$), i.e. the terms with negative (resp. positive) $Re\lambda_{ij}$, are to be kept.

For each layer, the coefficients $D_j$ are to be determined from mechanical and electrical conditions holding on the interfaces between the layers and on outer surfaces. It is assumed that the vertical displacements are very small so that the shape of the layers is preserved.

For every two neighboring layers $I$ and $II$ the following mechanical conditions expressing the continuity of the displacement field and the equilibrium of the normal pressures must
hold on the interface between the layers:

\[ u^I_i = u^II_i, \]  \hspace{1cm} (9)

\[ C^I_{i3kl} \frac{\partial u^I_i}{\partial x_k} - e^I_{ki3} \frac{\partial \phi^I_i}{\partial x_k} = C^II_{i3kl} \frac{\partial u^II_i}{\partial x_k} - e^II_{ki3} \frac{\partial \phi^II_i}{\partial x_k}. \]  \hspace{1cm} (10)

Electrical interface conditions express the continuity of the normal component of the electric displacement and the continuity of the tangent component of the electric field:

\[ \frac{\partial \phi^I}{\partial x_j} + e^I_{3jkl} \frac{\partial u^I_i}{\partial x_k} = \frac{\partial \phi^II}{\partial x_j} + e^II_{3jkl} \frac{\partial u^II_i}{\partial x_k}, \]  \hspace{1cm} (11)

\[ \frac{\partial \phi^I}{\partial x_1} = \frac{\partial \phi^II}{\partial x_1}. \]  \hspace{1cm} (12)

In the simplest case, the structure does not contact anything so that the following relations hold on the free surface of the top layer:

\[ C_{i3kl} \frac{\partial u^I_i}{\partial x_k} - e_{ki3} \frac{\partial \phi^I_i}{\partial x_k} = 0, \]  \hspace{1cm} (13)

\[ \epsilon_{3j} \frac{\partial \phi}{\partial x_j} + e_{3kl} \frac{\partial u_i}{\partial x_k} = 0, \]  \hspace{1cm} (14)

\[ \frac{\partial \phi}{\partial x_1} = 0. \]  \hspace{1cm} (15)

Note that the terms of the relations (10) – (15) involving the stress piezoelectric tensor disappear in the case of a purely dielectric layer. For a metallic layer, the terms containing the electric field vanish additionally.

Computing \( u_i \) and \( \phi \) with the use of (4) – (6), and (8) and substituting them into (9)– (15) for all the pairs of neighboring layers, we obtain a homogeneous system of linear equations for unknown coefficients \( D_j \). Denote by \( G(\omega, \kappa) \) the matrix of this system. Fixing the circular frequency \( \omega \) and denoting by \( V = \omega/\kappa \) the unknown wave velocity, we can consider \( G \) as a function of \( V \). The wave velocity is feasible if and only if the system has a nontrivial solution, which is equivalent to the condition: \( \det \left| G'(V)G(V) \right| = 0 \). The last
equation can be easily solved in $V$ because the computation of the left-hand-side runs very quickly even on an ordinary computer. Usually, three roots are being found whenever the layers are sufficiently thin, which corresponds to three types of waves propagating with different velocities.

2.2 Treatment of the contact with a fluid

Assume now that the upper surface of the top layer contacts with a viscous, weakly compressible fluid governed by the following linearized Navier-Stokes equations:

$$\rho_0 v_i t - \nu \Delta v_i - (\zeta + \frac{\nu}{3}) \frac{\partial}{\partial x_i} \text{div} \vec{v} + \frac{\partial}{\partial x_i} p = 0, \quad (16)$$

$$\alpha p_t + \rho_0 \frac{\partial}{\partial x_i} v_i = 0. \quad (17)$$

Here, $v_i, i = 1, 2, 3,$ are the components of the velocity field, $p$ is the static pressure, $\nu$ and $\zeta$ are the dynamic and volume viscosities of the fluid, respectively, $\rho_0$ is the initial density of the fluid and $\alpha = \partial \rho / \partial p$ expresses the compressibility of the fluid.

Instead of the condition (13) holding on the free surface of the top layer, we consider now the following matching conditions expressing the continuity of the velocities and the equilibrium of the normal pressures:

$$\frac{\partial u_i}{\partial t} = v_i, \quad (18)$$

$$C_{ikl} \frac{\partial u_i}{\partial x_k} - e_{kij} \frac{\partial \phi}{\partial x_k} = -p\delta_{i3} + \nu \left( \frac{\partial v_i}{\partial x_3} + \frac{\partial v_3}{\partial x_i} \right) + (\zeta - \frac{2}{3} \nu) \delta_{i3} \text{div} \vec{v}. \quad (19)$$

Look for the velocity in the form similar to (4) and for the pressure in the form like (5). Substitute this ansatz into equations (16) and (17) to arrive at a linear system like (6), (7). Solutions of such a system are of the form (8). Substitution them in the interface conditions (18) and (19) results in linear equations with respect to the arbitrary coefficients (compare with $D_j$ from section 2.1), which contributes to the matrix $G(V)$ (see section 2.1).
2.3 Treatment of the interface with a surrounding dielectric medium

The conditions (14) and (15) are not quite precise because they assume the absence of the electric field outside the solid structure, which is not true as a rule. To account for dielectric properties of the contacting medium, the conditions (14) and (15) must be replaced by:

\[
\epsilon_{3j} \frac{\partial \phi}{\partial x_j} + \epsilon_{3kl} \frac{\partial u_l}{\partial x_k} = \epsilon_0 \frac{\partial \hat{\phi}}{\partial x_j},
\]

(20)

\[
\frac{\partial \phi}{\partial x_1} = \frac{\partial \hat{\phi}}{\partial x_1}.
\]

(21)

where \(\epsilon_0\) is the dielectric permittivity of the surrounding medium and \(\hat{\phi}\) is the electric potential satisfying the equation:

\[
\epsilon_0 \frac{\partial^2 \hat{\phi}}{\partial x_i \partial x_j} = 0.
\]

(22)

2.4 Treatment of the interface between a bristle-like structure and a fluid

In many applications, the surface of the top layer contacting the fluid is covered with a very thin periodic bristle-like structure (see Figure 2). An example of such a structure is an aptamer-protein layer mentioned in section 1.

The height of the bristles is very small and their density is very high, which makes impossible a direct modeling of such a structure using fluid-solid interface conditions. Instead

![Fig. 2. Bristle-like solid-fluid interface.](image-url)
of that, the bristle-fluid structure is replaced by an averaged material whose properties are derived as the number of bristles goes to infinity whereas their thickness goes to zero, thereby the height of the bristles remains constant. Thus, we end up with a new layer whose thickness is equal to the height of the bristles and whose behavior is described by limiting equations derived using the following technique.

Let \( \Omega \subset \mathbb{R}^3 \) be the total domain of the bristle-fluid structure consisting of the domain \( \Omega_S \) occupied by bristles and \( \Omega_F \) occupied by a fluid. The interface separating these continua is denoted by \( \Gamma \). The structure is assumed to be periodic in \( x_1 \) and \( x_2 \) directions and independent of \( x_3 \). The basic governing equations read as follows:

\[
\begin{align*}
\rho_F \vec{v}_t - \text{div}P\vec{v}_x + \nabla p &= 0 \quad \text{in} \quad \Omega_F, \\
\alpha p_t + \rho_F \text{div}\vec{v} &= 0 \quad \text{in} \quad \Omega_F, \\
\rho_S \ddot{u}_t - \text{div}C\vec{u}_x &= 0 \quad \text{in} \quad \Omega_S.
\end{align*}
\]

Note that the equations describing the fluid part are the same as (16) and (17) rewritten in a tensor form. The no-slip and pressure equilibrium conditions on the fluid-solid interface read

\[
\begin{align*}
\vec{u}_t &= \vec{v} \quad \text{on} \quad \Gamma, \\
C\vec{u}_x \cdot \vec{n} &= \left( -p I + P\vec{v}_x \right) \cdot \vec{n} \quad \text{on} \quad \Gamma.
\end{align*}
\]

Here, \( \rho_F \) and \( \rho_S \) are the densities of the fluid and the solid parts, respectively; \( \vec{v} \) is the velocity field in the fluid, \( p \) is the pressure in the fluid, \( \vec{u} \) is the displacement field for the solid part. The coefficient \( \alpha \) characterizes the compressibility of the fluid. The fourth-rank tensors \( P \) and \( C \) express the viscosity of the fluid and the elastic stiffness of the solid, respectively.

To overcome the difficulty with the no-slip condition (26), the approach proposed by
J.-L. Lions in [8] is used. The basic idea is to use the velocity instead of the displacement in equation (25) as the state variable, which is done by means of the following integral operator:

\[ \mathcal{J}_t \vec{v} = \int_0^t \vec{v}(s) ds. \]

The equation (25) assumes then the form

\[ \rho_S \vec{v}_t - \text{div} C \mathcal{J}_t \vec{v}_x = 0, \]

where \( \vec{v} = \vec{u}_t \). The pressure \( p \) in (24) is expressed through the velocity \( \vec{v} \) as follows

\[ p = -\frac{\rho_F}{\alpha} \text{div} \mathcal{J}_t \vec{v}. \] (28)

Assume that the \( (x_1, x_2) \)-projection of the base cell of the bristle structure is a square \( \Sigma \) containing just one bristle (see Figure 3). Let \( \Sigma_{\delta} \) be the corresponding projection of the bristle and \( \Sigma_F = \Sigma \setminus \text{cl} (\Sigma_{\delta}) \).

\[ \Sigma_{\delta} \subseteq \Sigma_F \]

Fig. 3. Periodic cell \( \Sigma \).

Let \( \hat{\chi}(x_1, x_2) \) be the \( \Sigma \)-periodic extension of the characteristic function of the domain \( \Sigma_F \) to all \( \mathbb{R}^2 \). We define then the characteristic function of the domain occupied by fluid as follows:

\[ \chi^\varepsilon(\vec{x}) = \hat{\chi}(\frac{x_1}{\varepsilon}, \frac{x_2}{\varepsilon}), \] (29)

where \( \varepsilon \) is a refinement parameter. The value \( \varepsilon = 1 \) corresponds to the original structure, the bristles become finer and their density grows whenever \( \varepsilon \to 0 \).

Using (29), we can rewrite equations (23) – (25) as one equation with discontinues coef-
ficients in the whole domain Ω:

\[ \rho^\varepsilon \vec{v}^\varepsilon_t - \nabla \vec{M}^\varepsilon \vec{v}^\varepsilon_x = 0, \]  

(30)

where

\[ \rho^\varepsilon = \rho_F \chi^\varepsilon + \rho_s (1 - \chi^\varepsilon), \quad \vec{M}^\varepsilon = \chi^\varepsilon P + \left( \chi^\varepsilon P_F I \otimes I + (1 - \chi^\varepsilon)C \right) J_t. \]

The interface condition (26) is equivalent to the continuity of \( \vec{v}^\varepsilon \) on \( \Gamma \) but the condition (27) assumes now the form

\[ C J_t \vec{v}^\varepsilon_x \cdot \vec{n} = \left( \frac{\rho_F}{\alpha} \text{div} J_t \vec{v}^\varepsilon \cdot I + P \vec{v}^\varepsilon_x \right) \cdot \vec{n} \quad \text{on } \Gamma^\varepsilon, \]  

(31)

when accounting for (28).

As it is shown in [9] the sequence \( \{ \vec{v}^\varepsilon \} \) of solutions of (30) and (31) converges in the two-scale sense (see [10]) to a limiting function \( \vec{v} \) which satisfies the following limiting equation

\[ \rho_0 \vec{v}_t - \text{div} J \hat{C} \vec{v}_x - \text{div} \hat{P} \vec{v}_x - \text{div} \int_0^t \omega (t - s) \vec{v}_x(s) \, ds = 0. \]  

(32)

The term containing the tensor \( \hat{P} \) describes a viscous damping and originates from the fluid part of the bristle structure. The term containing the tensor \( \hat{C} \) represents elastic stresses. The tensor \( \hat{C} \) is degenerate, its kernel is such that volume preserving deformations (shear deformations in particular) do not produce elastic stresses. The integral term represents a memory effect that is responsible for viscoelastic properties of the limiting material. It is stated numerically that the memory effect is very weak. The system “forgets” the current history very quickly. Therefore, the integral term in the left-hand-side of (32) can be dropped. Note that the limiting material described by (32) is as a rule anisotropic even though the material of the solid part of the bristle structure is isotropic. This occurs because the limiting material inherits geometric properties of the bristle structure. The computation of the
tensors \( \hat{P}, \hat{C}, \) and \( \omega(\tau) \) is based on an analytical representation of solutions of the so-called cell equation which arises in homogenization theory. Numerical implementation of such an representation involves finite element methods.

Substituting \( \vec{u} = J \vec{v} \) results in the final equation for bristle-like layers:

\[
\rho u_{it,t} - \hat{C}_{ijkl} \frac{\partial^2 u_l}{\partial x_j \partial x_k} - \hat{P}_{ijkl} \frac{\partial^2 u_l}{\partial x_j \partial x_k} = 0.
\] (33)

The conditions at the interface with the underlying solid layer and the overlying fluid are similar to those described in sections 2.1 and 2.2, respectively, if we observe that the components of the pressure on the surface with the normal \((0,0,1)\) are expressed as:

\[
p_i = \hat{C}_{13kli} \frac{\partial u_l}{\partial x_k} + \hat{P}_{13kli} \frac{\partial u_l}{\partial x_k},
\] (34)

where \( i = 1,2,3 \). The form of (33) and (34) allows us to embed such structures into the general scheme described in section 2.1 quite easily.

3 Program description

The program has a standard windows users friendly interface written in Visual C++.

The usage of the program includes the following steps. First of all one composes a multi-layered structure and specifies its parameters including the excitation frequency \( \omega \) (see Figure 4). The structure can contain arbitrary number of layers of diverse types: solid, fluid, medium, or bristle. If a layer is anisotropic, the orientation of its material is described in terms of successive rotations of the reference system. Since some materials require a lot of parameters to be specified (up to 46 independent parameters for a piezoelectric anisotropic materials) an option supporting the import of material parameters from existing models is implemented. For bristle-like layers the geometry of the basic (structural) cell and the elastic
parameters of the solid material are specified. The coefficients of the limiting equation (33) are calculated then automatically by means of an embedded finite element subsystem.

After composing the structure and specifying the wave frequency, the fitting function is computed and graphically presented (see Figure 5). Now, the roots of the fitting function can be localized and found precisely. As one of the feasible wave velocities is found, the corresponding wave is defined and can be observed (see Figure 6). Moreover, the whole dispersion curve for the selected wave type (dependency between the wave velocity and the frequency) can be automatically computed and drawn. Such features make the program useful for researchers working on acoustic sensors. Note that the processing of bristle-like structures is a quite new option of our development. As far as we know, other programs (see e.g. [1], [3]) do not provide such a capability.
Fig. 5. Main window of the program. Roots of the fitting function are the feasible wave velocities for different wave types.

Fig. 6. Profiles of the electric potential, components of the displacement, and the electric field. The case of a shear wave (only $u_2 \neq 0$).
4 Simulation results

Results of testing the program are presented in Figures 7 and 8. Figure 7 shows a dispersion curve generated by the program for a simple structure consisting of a half-space isotropic substrate covered with an isotropic layer. For such a simple structure, an analytical solution exists (see [11]). The squares mark points found analytically. Figure 8 shows the velocity of the Rayleigh wave on X-cut $LiTaO_3$ substrate compared with results of [12].

Figures 9-11 are related to the development of a biosensor at the research center caesar (see [13,7,14,15]).

Fig. 7. Dispersion curve generated by the program and points found analytically.

Fig. 8. Velocity of the Rayleigh wave on X-cut of $LiTaO_3$: (a) paper [12], (b) simulation.
Fig. 9. Verification of physical experiments through numerical simulations: (a) experiment, (b) simulation.

Figure 9 demonstrates the verification of physical experiments connected with the determination of the biosensor sensitivity. A 9 nm copper film is deposited on the top layer of the biosensor. Curve (a) shows the time performance of the etching of the cooper film. The water flux is being alternated with the flux of an acid solution. The phase shift is being measured. Curve (b) represents the phase shift computed using dispersion relations. The simulation proves the assumption that the jump at the acid-to-water transition is caused by the change of the fluid viscosity.

The most important application of the developed tools is the estimation of the mass sensitivity of a Love wave sensor (see section 1) against proteins. The structure under study is shown in Figures 1 and 4. The adhered protein is represented as a bristle-like layer located on the top of the bristle-like aptamer layer. The sensitivity is expressed by:

\[
\frac{\omega_0}{\rho h \tau} \left( \frac{l_1}{l_0} - \frac{l}{l_0} \right),
\]

where \( l \) is the travel distance of the wave, \( \tau = |\Sigma_S|/(|\Sigma_S| + |\Sigma_F|) \) (see Figure 3) is the
packing density of the protein molecules, \( h \) is the molecular diameter of the protein, i.e. the thickness of the protein layer, \( \rho \) is the density of the protein, \( \omega_0 \) is the operation frequency of the biosensor, and \( V_1 \) and \( V_0 \) are the wave velocities in the case of the presence and absence of the adhered protein layer, respectively.

The operation frequency of the biosensor is calculated from a prescribed wave length using the following iterating relation:

\[
\omega_{n+1} = \frac{2\pi V(\omega_n)}{\lambda},
\]

where \( \lambda = 28\mu m \) is the optimal operation wave length determined from some technical reasons. The dependence \( V(\omega) \) is calculated by means of the developed program.

Figures 10 and 11 show computed dependencies of the sensitivity of the sensor on the thickness of the guiding layer. In Figure 10, a comparison for two different cuts of a quartz crystal used as the substrate material is done. Figure 11 presents sensitivity curves for different thicknesses of the gold shielding layer. Relative high values of the sensitivity comparing

![Fig. 10. Dependence of the sensitivity of the Love wave sensor on the thickness of the guiding layer:](image)

(a) ST-cut quartz substrate, (b) CT-cut quartz substrate.
Fig. 11. Sensitivity curves depending on the thickness of the shielding layer: (a) 40 nm, (b) 100 nm, (c) 200 nm. with values obtained through the adding of a solid layer which simulates the protein film are in very good agreement with experiments.

5 Conclusion

The paper describes a numerical method and a program for the computation of dispersion relations for surface and bulk acoustic waves in multi-layered anisotropic structures that may contain specific bristle-like layers contacting with fluids. In view of biosensoric applications, the most important features of the method are accounting for piezoelectric properties of materials, ability to operate with very thin layers, and an adequate treatment of the interface between solid bristle-like structures and fluids.

The method presented in this paper has already been successfully applied to the development of a biosensor at the research center caesar.

Sometimes, investigated structures contain very much of periodically alternating layers whose thickness is significantly less than the wave length. Such a sandwich can be replaced
by an averaged layer whose properties can be derived by means of the homogenization technique. Note that the homogenization will be performed in the vertical direction only. Therefore, explicit formulas for parameters of the averaged material can be obtain. This option is expected to be implemented.

References


